Math 1320 Practice Exam 1 Fall 2017

1) A manufacturer sells car batteries for $150 each. The company's fixed costs are $45,000 per month, and marginal costs are $55 per battery.

   a) Write the equations for the revenue, cost, and profit functions. Let x be the number of batteries.

   $R(x) =$

   $C(x) =$

   $P(x) =$

   b) How many batteries must be sold to break even? Round to the nearest battery.

2) Let $f(x) = 2x^2 - 3x + 1$

   a) Calculate $f(-3)$

   b) Calculate $f(2) - f(-2)$

   a) Find and simplify $f(x+h)$

   d) Find and simplify $\frac{f(x+h)-f(x)}{h}$

3) Find the equation of the line that passes through the points $(-2, 1)$ and $(2, 3)$.

4) The XYZ Widget factory can produce 80 widgets in a day at a total cost of $8,000 and it can produce 100 widgets a day at a total cost of $10,000.

a) What are the company's daily fixed costs and marginal cost per widget?

b) Use the cost function to estimate the cost of manufacturing 400 widgets in a day.

5) You can sell 100 pet rocks per week if they are marked at $1 each, but only 40 each week if they are marked at $2 per rock. Your rock supplier is prepared to sell you 30 rocks each week if they are marked at $1/rock, and 120 each week if they are marked at $2 per rock.

   a) Write down the associated linear demand and supply functions.

   b) At what price should the rocks be marked so that there is neither a surplus nor a shortage of rocks?

6) Sketch the graph of the quadratic function, indicating the coordinates of the vertex, the y-intercept, and the x-intercepts (if any).

$$f(x) = -x^2 + 4x - 4$$

7) The demand function for a specific product is given by $q = 60 - \frac{1}{3}p$ units, where p is the price per unit.
   a) Find the revenue function $R(p)$.
   b) Find the price that maximizes the revenue.
   c) Find the maximum revenue.
   d) How many units must be produced to maximize the revenue?

8) Actinium is a highly radioactive element. The most common isotope of actinium is produced as a by-product in nuclear reactors, and has a half-life of 21.77 years.

   a) Obtain an exponential decay model for actinium-227 in the form $Q(t) = Q_0 e^{-kt}$. (Round k to four decimal places.)
   b) About 20 milligrams of actinium are produced in a certain nuclear reactor. Use your model to predict how long it will take for this amount of actinium to decay to one milligram.

9) The half-life of cobalt 60 is 5 years.
   a) Obtain an exponential model for cobalt 60 in the form $Q(t) = Q_0 e^{-kt}$. (Round coefficients to three significant digits.
   b) Use your model to predict, to the nearest year, the time it takes for one third of the sample of cobalt 60 to decay.

10) Find the equation of the exponential function passing through the points $(2, 9)$ and $(4, 20.25)$.

11) A bacteria culture starts with 2,500 bacteria at time t = 0. Two hours later there are 13,500 bacteria. Round your values to two decimal places as necessary.
   a) Find an exponential model for the size of the culture as a function of time t in hours.
   b) Use the model to predict how many bacteria there will be after 3 hours.

12) There were 3,500 bacteria in a Petri dish (at time $t = 0$ hours). Four hours later, there were 5,500 bacteria in the dish. Find the mathematical model that represents the number of bacteria after t hours. It's an exponential formula of the form $Q(t) = Q_0 e^{kt}$.
   Round k to 4 decimal places. Include the units in the answer.