# Math 1320 Practice Exam 1 Fall 2018

1) fixed cost \$70 marginal cost 40¢ per copy sells for 50¢ per copy

a) $C(x) = 0.40x + 70$

$R(x) = 0.50x$ $\qquad R - C = 0.50x - (0.40x + 70)$

$P(x) = 0.10x - 70$ $\qquad 0.50x - 0.40x - 70$

b) profit or loss ⇒ use $P(x)$ at 500 copies ⇒ $x = 500$

$P(500) = 0.10(500) - 70 = -20$ loss of \$20

c) Break-even ⇒ $P = 0$ OR $R = C$

$$0 = 0.10x - 70 \qquad\qquad 0.50x = 0.40x + 70$$

$$\underline{+70 \qquad\quad +70} \qquad\qquad \underline{-0.40x \qquad -0.40x}$$

$$\frac{70}{0.10} = \frac{0.10x}{0.10} \qquad\qquad \frac{0.10x}{0.10} = \frac{70}{0.10}$$

either method not both

700 copies $= x$ $\qquad\qquad$ $x = 700$ copies

2) $f(x) = x^2 + 3x + 1$ $\qquad$ Skeleton $f(\ ) = (\ )^2 + 3(\ ) + 1$

a) $f(0) = (0)^2 + 3(0) + 1 = \boxed{1}$

b) $f(-1) = (-1)^2 + 3(-1) + 1 = 1 - 3 + 1 = \boxed{-1}$

c) $f(a) = (a)^2 + 3(a) + 1 = a^2 + 3a + 1$

d) $f(x+h) = (x+h)^2 + 3(x+h) + 1$ $\qquad$ now simplify

$= (x+h)(x+h) + 3(x+h) + 1$

$= x^2 + \underline{xh + xh} + h^2 + 3x + 3h + 1$

$= x^2 + 2xh + h^2 + 3x + 3h + 1$

3) Line: $y = mx + b$ points $(-2, 1)$ and $(2, 3)$

A) find $m = \dfrac{3 - 1}{2 - (-2)} = \dfrac{2}{4} = \dfrac{1}{2}$

B) Use a point and m to find b

$3 = \frac{1}{2}(2) + b$

$3 = 1 + b$

$2 = b$

c) Write eq: $\boxed{y = \frac{1}{2}x + 2}$

4) 80 widgets cost \$8000

100 widgets cost \$10,000

a) daily fixed and marginal ⇒ find b and m

(80, 8000) (100, 10000)

*assume linear?

$m = \dfrac{10,000 - 8,000}{100 - 80} = \dfrac{2000}{20} = \boxed{100}$ marginal cost per widget

$10,000 = 100(100) + b$

$10,000 = 10000 + b$

$0 = b$ \$0 fixed cost

b) cost function ⇒ $C(x) = 100x + 0$ 400 widgets ⇒ $x = 400$

$C(400) = 100(400) + 0 = 100(400) = \$40,000$

5) sell 100 rocks \$1 each 40 rocks \$2/each demand (1, 100) (2, 40)

supply 30 at \$1 each 120 at \$2 each Supply (1, 30) (2, 120)

always (price, quantity)

a) Demand $m = \dfrac{40 - 100}{2 - 1} = \dfrac{-60}{1}$

Supply $m = \dfrac{120 - 30}{2 - 1} = \dfrac{90}{1}$

$100 = -60(1) + B$

$100 = -60 + B$

$160 = B$

$q = -60p + 160$

$30 = 90(1) + b$

$-60 = b$

$q = 90p - 60$

b) ... ⇒ equilibrium

$-60p + 160 = 90p - 60$

$+60p \quad +60 \quad +60p \quad +60$

$220 = 150p$

$\dfrac{220}{150} = p = \$1.47$

1.466666 rounded

6) a) linear demand $q(p)$ so (price, quantity)

$(325, 1110)$ $(245, 1910)$

$m = \frac{1910 - 1110}{245 - 325} = \frac{800}{-80} = -10$

$1910 = -10(245) + b$

$1910 = -2450 + b$

$4360 = b$

$q(p) = -10p + 4360$

b) price is \$375 $\Rightarrow$ $q(375) = -10(375) + 4360 = 610$ million phones

c) For every \$1 increase in price, sales of cell phones decrease by __$10$__ units.

(this is what slope means)

7) $f(x) = -x^2 + 4x - 4$

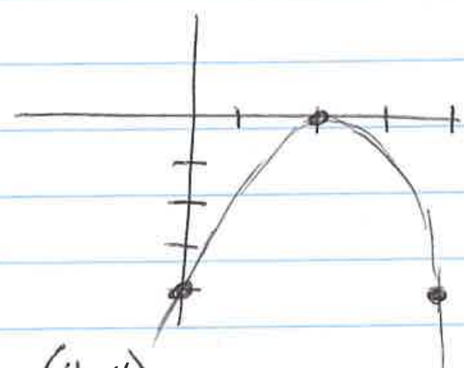
vertex: $x = \frac{-b}{2a} = \frac{-(4)}{2(-1)} = \frac{-4}{-2} = 2$

$(2, 0)$

$y = -(2)^2 + 4(2) - 4 = -4 + 8 - 4 = 0$

y-intercept: $(0, -4)$

vertex is x-intercept so symmetric point is $(4, -4)$



8) demand is $q = -4p + 480$

a) At what price (find $p$) largest Revenue $\Rightarrow$ make revenue function!

$R = \text{price} * \text{quantity} = pq = p(-4p + 480)$ so

$R(p) = -4p^2 + 480p$ now largest implies vertex

$p = \frac{-b}{2a} = \frac{-480}{2(-4)} = \frac{-480}{-8} = \$ = \boxed{\$60}$

b) what is largest revenue $\Rightarrow$ Revenue at price of \$60

$R(60) = -4(60)^2 + 480(60)$

$= -14400 + 28800$

$= \boxed{\$14{,}400}$

oops! I got $R = 0$ so I knew I messed up on price!

9) Half-life $t_h = 21.77$

a) $Q(t) = Q_0 e^{-kt}$ find $k$ to 4 decimals

$k \cdot t_h = \ln(2)$ so $k = \dfrac{\ln(2)}{t_h} = \dfrac{\ln(2)}{21.77} \approx 0.0318$

no initial quantity given so answer is

$Q(t) = Q_0 e^{-0.0318t}$

b) If $Q_0 = 20$, find $t$ to get $Q = 1$

$1 = 20e^{-0.0318t}$

isolate: $\dfrac{1}{20} = e^{-0.0318t}$

switch: $-0.0318t = \ln\left(\frac{1}{20}\right)$

solve: $t = \dfrac{\ln\left(\frac{1}{20}\right)}{-0.0318} \approx 94.205$ about $\boxed{94 \text{ years!}}$

10) half-life $t_h = 5$

a) $k = \dfrac{\ln(2)}{t_h} = \dfrac{\ln(2)}{5} = 0.139$  $Q(t) = Q_0 e^{-0.139t}$

b) 1/3 to decay means 2/3 remains so whatever $Q_0$ is,
$Q = \frac{2}{3}Q_0$ that is, solve

$\frac{2}{3}Q_0 = Q_0 e^{-0.139t}$

isolate: $\dfrac{\frac{2}{3}Q_0}{Q_0} = \dfrac{Q_0 e^{-0.139t}}{Q_0}$

$\frac{2}{3} = e^{-0.139t}$

switch forms: $-0.139t = \ln(2/3)$

solve: $t = \dfrac{\ln(2/3)}{-0.139} \approx 2.917$

about $\boxed{3 \text{ years}}$

11) triples every 9 months

a) Find b in $y = Ab^t$ for this situation

- triples indicates $\times 3$ <u>but</u>
- every 9 months to triple so $b = 3^{1/9} \approx 1.12983$

$b = 1.13$

b) Doubling time $2A = A(1.13)^t$

Isolate $2 = 1.13^t$

Switch $t = \log_{1.13}(2) = \dfrac{\log(2)}{\log(1.13)} \approx 5.671417$

5.7 months

12) 3500 at $t=0 \Rightarrow Q_0 = 3500$

4 hrs $\rightarrow$ 5,500 bacteria specific $t + Q$

$\dfrac{5500}{3500} = \dfrac{3500e^{k(4)}}{3500}$ to find k

isolate:

$\dfrac{11}{7} = e^{4k}$

Switch: $4k = \ln(11/7)$

Solve: $k = \dfrac{\ln(11/7)}{4} \approx 0.11299628 = 0.1130$

Model: $Q(t) = 3500e^{-0.1130t}$ bacteria after t hours