Limits at Infinity

By Tuesday J. Johnson



Suggested Review Topics

- Algebra skills reviews suggested:
 - None
- Trigonometric skills reviews suggested:
 - None

Applications of Differentiation

Limits at Infinity

Note

 There is a video that accompanies this section on my website and on YouTube. The link is: <u>https://www.youtube.com/watch?v=WKsDma</u> <u>L8YAM</u>

• In this video I talk specifically about Limits at Infinity and give several examples.

Definition of Limits at Infinity

- Let *L* be a real number.
 - 1. The statement $\lim_{x\to\infty} f(x) = L$ means that for each $\varepsilon > 0$ there exists an M > 0 such that $|f(x) L| < \varepsilon$ whenever x > M.
 - 2. The statement $\lim_{x \to -\infty} f(x) = L$ means that for each $\varepsilon > 0$ there exists an N < 0 such that $|f(x) L| < \varepsilon$ whenever x < N.

Definition of Horizontal Asymptote

The line y = L is a horizontal asymptote of the graph of f if either

$$\lim_{x\to\infty}f(x)=L$$

or

$$\lim_{x \to -\infty} f(x) = L$$

(That is, one limit or the other limit or both.)

Theorem 3.10 (Limits at Infinity)

If *r* is a positive rational number and *c* is any real number, then $\lim_{x\to\infty} \frac{c}{x^r} = 0$.

Furthermore, if x^r is defined when x < 0, then $\lim_{x \to -\infty} \frac{c}{x^r} = 0$.

Guidelines for Finding Limits at \pm or Rational Functions

- 1. If the degree of the numerator is less than the degree of the denominator, then the limit of the rational function is 0. (That is, the horizontal asymptote is the equation y = 0.)
- 2. If the degree of the numerator is equal to the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
- 3. If the degree of the numerator is greater than the degree of the denominator, then the limit of the rational function does not exist.

1.
$$\lim_{x \to (-\infty)} (\frac{5}{x} - \frac{x}{3})$$

To find this limit, find the individual limits:

$$\lim_{\substack{x \to (-\infty) \\ x \to (-\infty)}} \frac{5}{x} = 0$$
$$\lim_{x \to (-\infty)} \frac{x}{3} = DNE$$

Now the arithmetic becomes 0 - DNE = DNE. That is, this limit does not exist.

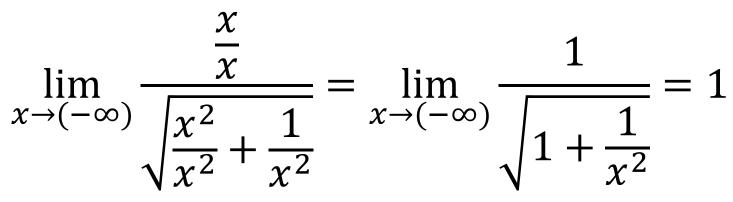
2.
$$\lim_{x \to \infty} \frac{x^2 + 3}{2x^2 - 1}$$

Since x is going to infinity, and is not zero, we can divide each term of the function by the highest power of x in the entire expression:

$$\lim_{x \to \infty} \frac{\frac{x^2}{x^2} + \frac{3}{x^2}}{\frac{2x^2}{x^2} - \frac{1}{x^2}} = \lim_{x \to \infty} \frac{1 + \frac{3}{x^2}}{2 - \frac{1}{x^2}} = \frac{1}{2}$$

3.
$$\lim_{x\to(-\infty)} \frac{x}{\sqrt{x^2+1}}$$

Notice that $x = \sqrt{x^2}$ so inside the radical we divide by x^2 and outside the radical we divide by x .



4.
$$\lim_{x\to\infty} \cos\frac{1}{x}$$

As
$$x \to \infty$$
, $\frac{1}{x} \to 0$. Therefore,

$$\lim_{x \to \infty} \cos \frac{1}{x} = \cos 0 = 1$$

Examples: Find the limit. You try it.

5.
$$\lim_{x \to \infty} \frac{3x^3 + 2x - 7}{\sqrt{x^4 + 5x}}$$

6.
$$\lim_{x \to (-\infty)} \frac{\sqrt{x^8 - 1}}{x^5 - 1}$$

End of Lecture