Section 3.8 Newton's Method:

A technique for approximating the real zeros of a Function.

What is the derivative of a function?

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$f'(x_1) = m$; *slope* of the tangent line at some x_1.

Using the point slope formula: $y-y_1=m(x-x_1)$

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$$y = f(x_1) + f'(x_1)(x - x_1)$$

Let y = 0, since we are looking for the zeros of the function.

$$0 = f(x_1) + f'(x_1)(x - x_1)$$

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Distribute $f'(x_1)$

$$0 = f(x_1) + f'(x_1)x - f'(x_1)x_1$$

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$$0 = f(x_1) + f'(x_1)(x - x_1)$$

$$0 = f(x_1) + f'(x_1)x - f'(x_1)\, x_1$$

Isolate $f'(x_1)x$

$$f'(x_1)x = f'(x_1)\, x_1 - f(x_1)$$

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$$f'(x_1)x = f'(x_1)\, x_1 - f(x_1)$$

Solve for *x*

$$x = [f'(x_1) / f'(x_1)]\, x_1 - [\, f(x_1) / f'(x_1)]$$

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$$x = [f'(x_1) / f'(x_1)]\, x_1 - [\, f(x_1) / f'(x_1)]$$

$$x = x_1 - f(x_1) / f'(x_1)$$

- If we do this again and again we have the process which is called Newton’s Method.

Newton's Method for Approximating the Zeros of a Function:

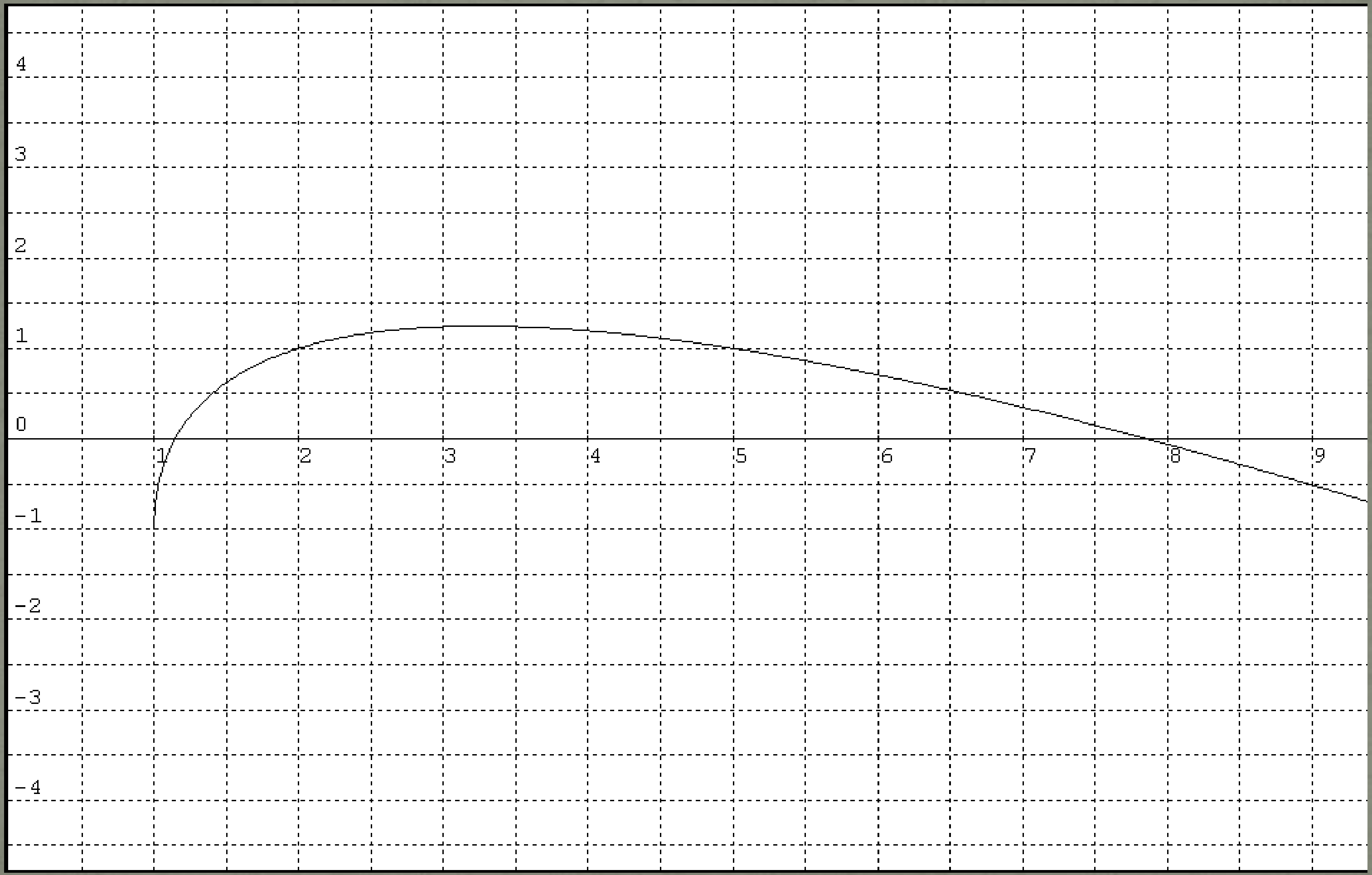
Let $f(c) = 0$, where f is differentiable on an open interval containing c. Then, to approximate c, use the following steps.

1. Make an initial estimate that is close to c. (A graph is helpful)
2. Determine a new approximation

   $x_{n+1} = x_n - f(x_n)/ f'(x_n)$
3. If $|x_n - x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to Step 2 and calculate a new approximation.

Each successive application of this procedure is called an **iteration**.

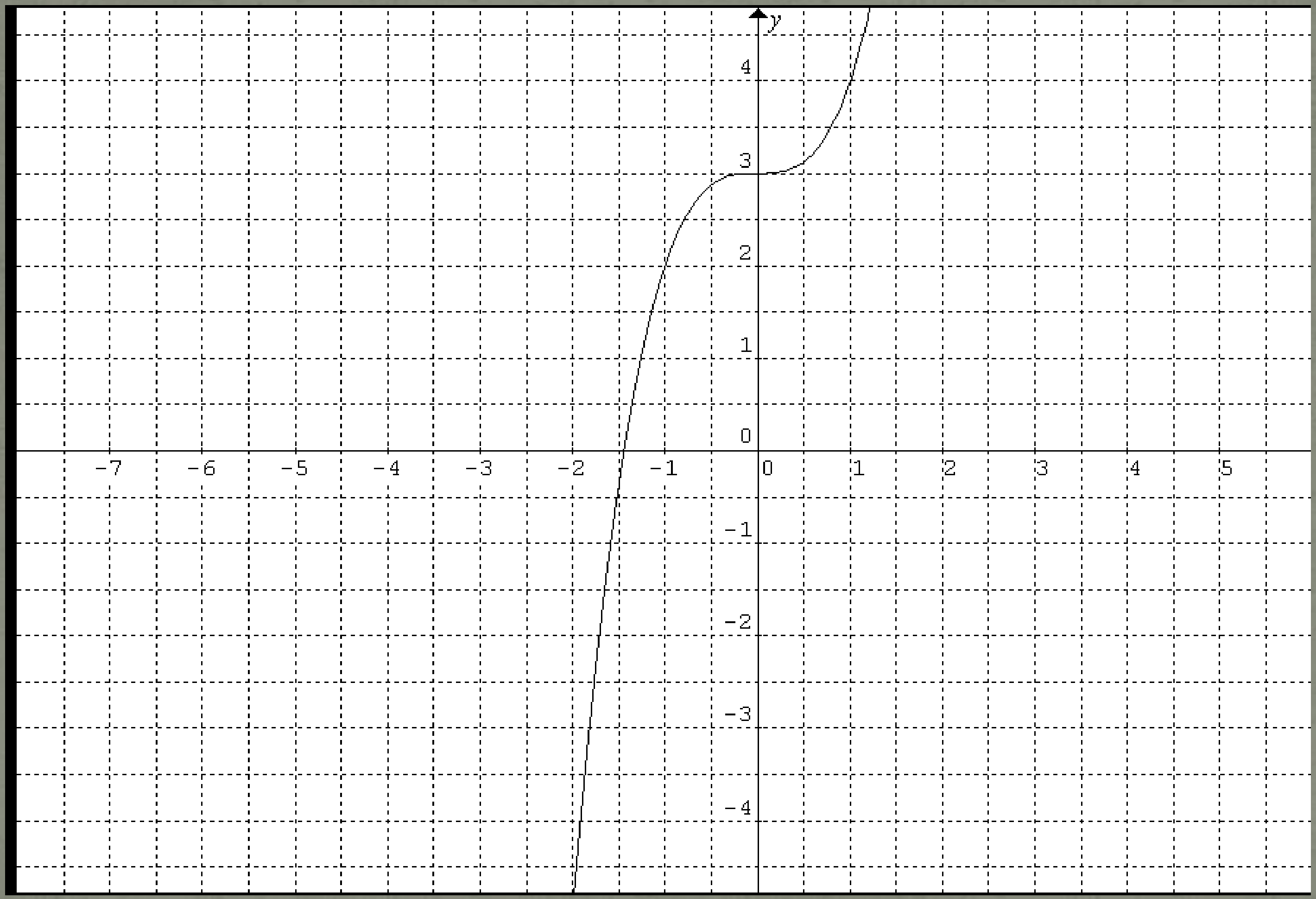
Graph of $f(x) = 3(x-1)^{1/2}-x$



Newton's Method: $f(x) = 3(x-1)^{1/2}-x$

Problem 7	x_n	$f(x_n)$	$f'(x_n)$	$f(x_n)/f'(x_n)$	$x_n-f(x_n)/f'(x_n)$		
$f(x)=3^*(x-1)^{1/2}-x$	1.1000	-0.1513	3.7434	-0.0404	1.1404		
	1.1404	-0.0162	3.0029	-0.0054	1.1458		
	1.1458	-0.0002	2.9280	-0.0001	1.1459		
	1.1459	0.0000	2.9271	0.0000		x =	1.1459
	7.1000	0.3095	-0.3927	-0.7881	7.8881		
	7.8881	-0.0145	-0.4285	0.0339	7.8542		
	7.8542	0.0000	-0.4271	0.0001	7.8541		
	7.8541	0.0000	-0.4271	0.0000		x =	7.8541

Graph of $f(x) = x^3+3$



Newton's Method: *f(x)* = x^3+3

Problem 9	**x_n**	**f(x)**	**f '(x_n)**	**f(x_n)/f '(x_n)**	**x_n-f(x_n)/f '(x_n)**		
f(x)=x^3+3	-2.0000	-5.0000	12.0000	-0.4167	-1.5833		
	-1.5833	-0.9693	7.5208	-0.1289	-1.4544		
	-1.4544	-0.0768	6.3463	-0.0121	-1.4424		
	-1.4424	-0.0006	6.2411	-0.0001		x =	-1.4424