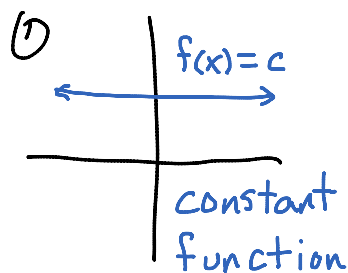


1.6 A Library of Parent Functions

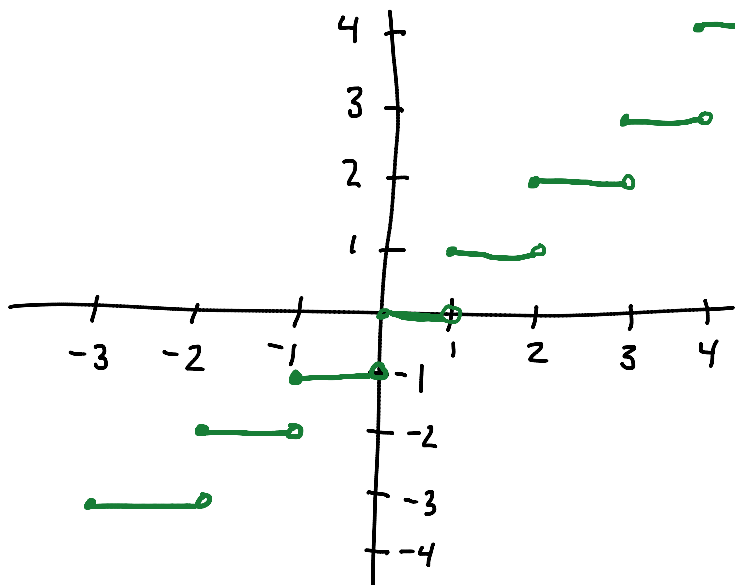
There are some functions that you just need to know without taking too long to think. We will look at the basic function and then describe how to transform them to other places on the graph. These eight functions are important to know:

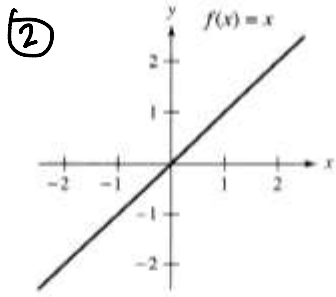
1. The constant function $f(x) = c$.
2. The identity function $f(x) = x$
3. The absolute value function $f(x) = |x|$.
4. The square root function $f(x) = \sqrt{x}$
5. The quadratic function $f(x) = x^2$
6. The cubic function $f(x) = x^3$
7. The reciprocal function $f(x) = \frac{1}{x}$
8. The greatest integer function $f(x) = \lfloor x \rfloor$

Graphs for 2 \rightarrow 7 can be found on the next page.

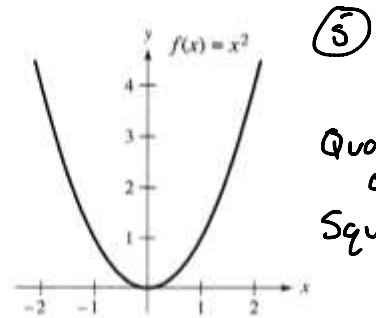


$f(x) = \lfloor x \rfloor$
greatest integer



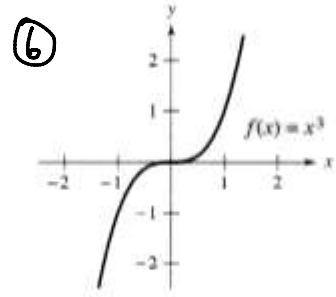


Identity function

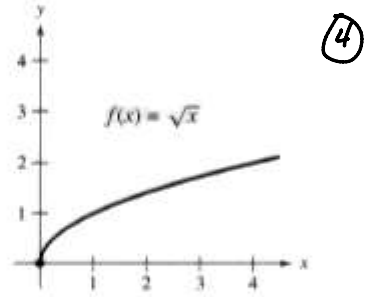


Quadratic
or
Squaring

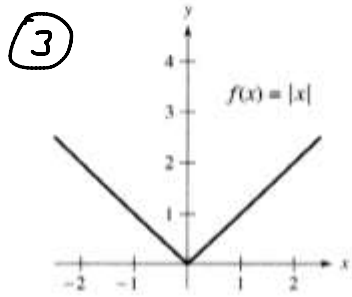
Squaring function



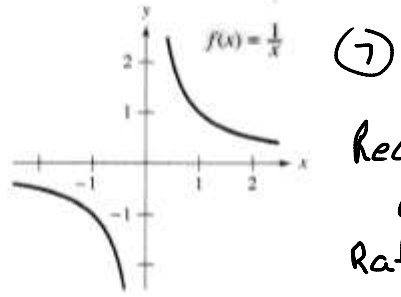
Cubing function



Square root function



Absolute value function



Reciprocal
or
Rational

Rational function

Let's explore this greatest integer function; it is the most uncommon of the bunch. The greatest integer operator, $\|x\|$, looks like absolute value but it is not. This represents the greatest integer less than or equal to x . That is, the result is always an integer (positive or negative counting numbers along with 0). If the input is an integer, the output is the same integer. So $\|2\| = 2$ and $\|-5\| = -5$. When it comes to non-integer inputs we need to slow down and think about it for a second. When I think of the greatest integer less than, I think left of. In this way I can rephrase in terms of a number line that the greatest integer of any input is the largest integer to the left of the input. For positive inputs we essentially just ignore the decimal portion: $\|3.7\| = 3$, $\|4.1\| = 4$, and $\|125.78932\| = 125$. However for negative inputs we need to think less than, left of, so we get: $\|-2.9\| = -3$, $\|-11.3\| = -12$, and so on.

Now that we have several different types of functions, we can combine pieces of them to make piece-wise defined functions.

Examples: Graph the piece-wise defined functions.

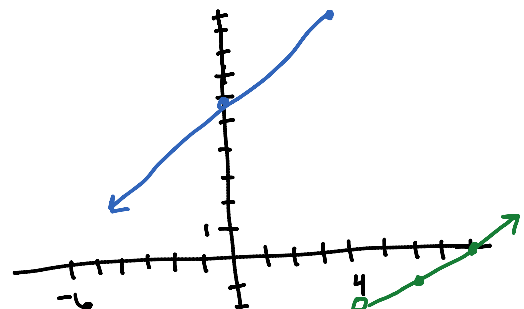
$$1. f(x) = \begin{cases} x+6 & x \leq 4 \\ \frac{1}{2}x-4 & x > 4 \end{cases}$$

| $x \leq 4$ | $x+6$ |
|------------|-------|
| 4 | 10 |
| 3 | 9 |
| 2 | 8 |
| 1 | 7 |
| 0 | 6 |

↑
a solid dot because $x \leq 4$ "or equal to"

| $x > 4$ | $\frac{1}{2}x-4$ |
|---------|------------------|
| 4 | -2 |
| 5 | ugh |
| 6 | -1 |
| 7 | ugh |
| 8 | 0 |

↑
open dot because $x > 4$ and not equal



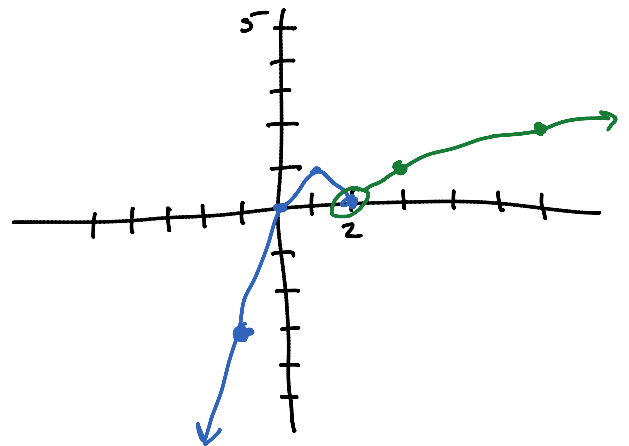
Note: the graph must still pass the VLT as it is the graph of a function

The cut value of $x=4$ is the only possibility of open or closed dots. All other points are solid.

$$2. f(x) = \begin{cases} 1 - (x-1)^2 & x \leq 2 \quad \leftarrow \\ \sqrt{x-2} & x > 2 \quad \leftarrow \end{cases}$$

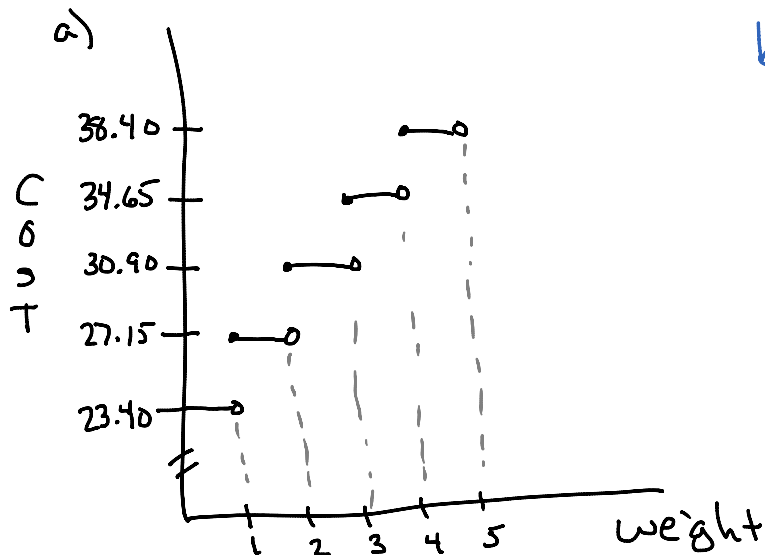
| $x \leq 2$ | $1 - (x-1)^2$ |
|------------|---------------|
| 2 | 0 |
| 1 | 1 |
| 0 | 0 |
| -1 | -3 |

| $x > 2$ | $\sqrt{x-2}$ |
|---------|--------------|
| 2 | 0 |
| 3 | 1 |
| 6 | 2 |
| 11 | 3 |



Example: The cost of sending an overnight package from Los Angeles to Miami is \$23.40 for a package weighing up to but not including 1 pound and \$3.75 for each additional pound or portion of a pound. A model for the total cost C (in dollars) of sending the package is $C = 23.40 + 3.75 \lceil x \rceil$, $x > 0$, where x is the weight in pounds.

- Sketch a graph of the model.
- Determine the cost of sending a package that weighs 9.25 pounds.



$$\begin{aligned} b) C &= 23.40 + 3.75 \lceil 9.25 \rceil \\ &= 23.40 + 3.75(9) \\ &= 23.40 + 33.75 \\ &= 57.15 \end{aligned}$$

Example: A mechanic is paid \$14.00 per hour for regular time and time-and-a-half for overtime. The

weekly wage function is given by $W(h) = \begin{cases} 14h & 0 < h \leq 40 \\ 21(h-40) + 560 & h > 40 \end{cases}$ where h is the number of hours

worked in a week.

a) Evaluate $W(30)$, $W(40)$, $W(45)$, and $W(50)$.

$$W(30) = 14(30) = \$420$$

$$W(40) = 14(40) = \$560$$

$$W(45) = 21(45-40) + 560 = 21(5) + 560 = \$665$$

$$W(50) = 21(50-40) + 560 = 21(10) + 560 = \$770$$

b) The company increased the regular work week to 45 hours. What is the new weekly wage function?

$$W(h) = \begin{cases} 14h, & 0 \leq h \leq 45 \\ 21(h-45) + 630, & h > 45 \end{cases}$$

$$14(45) = 630$$