Chapter Three: Exponential and Logarithmic Functions

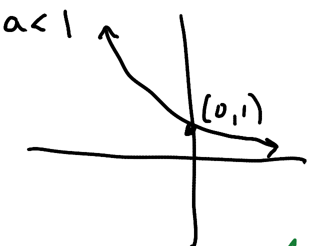
3.1 Exponential Functions and Their Graphs

Definition of Exponential Function – The exponential function *f* with base '*a*' is denoted by $f(x)=a^x$ where $a>0, a\neq 1,$ and x is any real number.

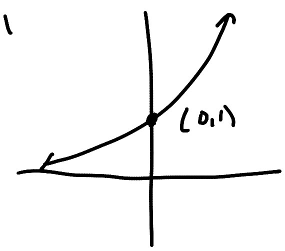
domain

Fact: The graph of $f(x)=a^x$ has one of two basic forms. If $0<a<1$, the graph is decreasing and if $a>1$, the graph is increasing. It has y-intercept (0, 1) and is a 1-1 monotonic function. The domain is all real numbers and the range is all *y > 0*. For $0<a<1$, we frequently think of a horizontal rotation and refer to it as $f(x)=a^{-x},\ a>1$.

$0<a<1$



$a>1$



monotonic means 1-1
so exponential functions have inverses!

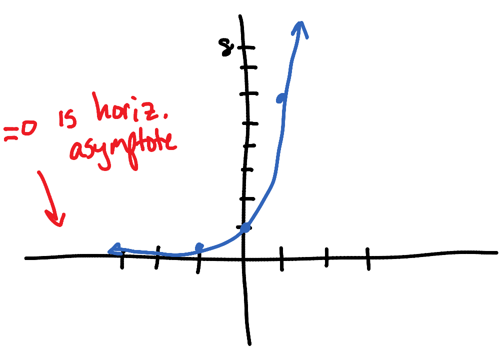
Knowing the basic shape, we can now transform the graph using the concepts from chapter 1.

Examples: Graph the function. Label at least three points with exact values.

1. $f(x)=6^x$

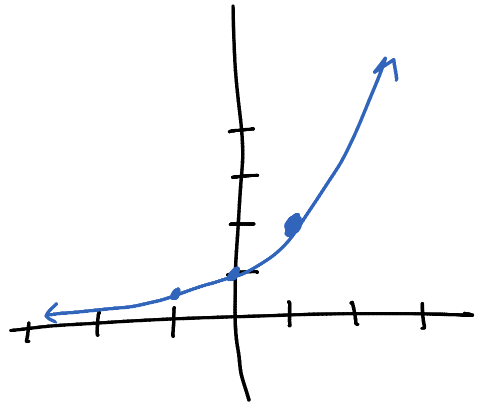
x	y
-1	$6^{-1}=\frac{1}{6}$
0	$6^0=1$
1	$6^1=6$

y=0 is horiz. asymptote



2. $g(x)=\left(\frac{1}{2}\right)^{-x} = 2^x$

x	y
-1	$\left(\frac{1}{2}\right)^{-(-1)}=\left(\frac{1}{2}\right)^{1}=\frac{1}{2}$
0	$\left(\frac{1}{2}\right)^{-(0)}=1$
1	$\left(\frac{1}{2}\right)^{-(1)}=\frac{2}{1}=2$

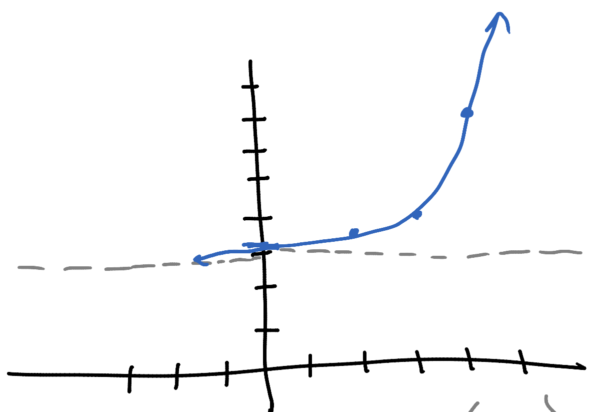


3. $h(x)=4^{x-3}+3$ ← new horizontal asymptote

right 3 up 3

The usual values moved right 3.

x	y
2	$4^{2-3}+3=4^{-1}+3=\frac{1}{4}+3=3.25$
3	$4^{3-3}+3=4^{0}+3=1+3=4$
4	$4^{4-3}+3=4^{1}+3=7$
0	$4^{0-3}+3=4^{-3}+3=\frac{1}{64}+3=3.015625$



range is $(3,\infty)$

Examples: Use the graph of *f* to describe the transformation that yields the graph of *g*.

1. $f(x)=3^x,\ g(x)=3^x+1$

graph of f(x) shifted up 1 unit

2. $f(x)=10^x,\ g(x)=10^{-x+3} = 10^{-(x-3)}$

graph of f(x) reflected about y-axis (horiz) and shifted right 3 units

Notice the simplification required before determining left or right.

Many times, the best base to use is the irrational number *e* $\approx$ *2.718281828…..* This number is called the natural base (because it is natural for mathematicians and scientists to use it). The function given by $f(x) = e^x$ is called the natural exponential function. When working with the natural base, do NOT use the decimal approximation; always use the value of *e* stored in your scientific calculator. Notice that since *e* > *1,* we know what its graph will look like.

The number e is named after the mathematician Leonard Euler.

Formulas for Compound Interest – After *t* years, the balance *A* in an account with principal *P* and annual interest rate *r* (in decimal form) is given by the following formulas.

1. For *n* compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ → monthly, $n = 12$; weekly, $n = 52$; annually, $n = 1$; daily, $n = 365$; quarterly, $n = 4$; etc.

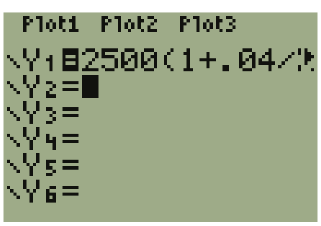
2. For continuous compounding: $A = Pe^{rt}$

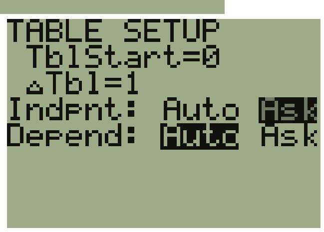
This formula comes from letting n get really large in formula 1.

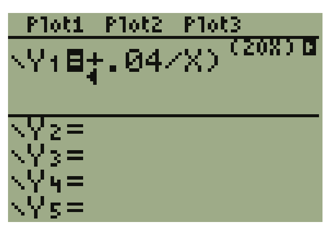
Examples: Complete the table to determine the balance *A* for \$2500 invested at 4% and compounded *n* times per year for 20 years.

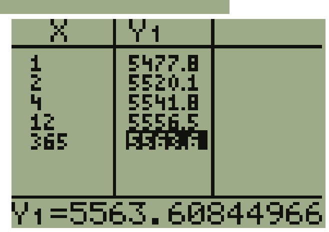
n	1	2	4	12	365	Continuous
A \$	5477.81	5520.10	5541.79	5556.46	5563.61	5563.85

$A = 2500e^{(.04 \times 20)}$









Example: The number V of computers infected by a computer virus increases according to the model $V(t) = 100e^{4.6052t}$, where t is the time in hours. Find the number of computers infected after (a) 1 hour, (b) 1.5 hours, and (c) 2 hours.

$V(1) = 100\, e^{(4.6052(1))} = 10{,}000$

$V(1.5) = 100\, e^{(4.6052(1.5))} = 100{,}004$

$V(2) = 100\, e^{(4.6052(2))} = 1{,}000{,}060$ rounded

That's some serious growth!