3.2 Logarithmic Functions and Their Graphs

Definition of Logarithmic Function with Base *a* – For *x > 0, a > 0,* and *a* $\neq$ *1,* $y = \log_a x$ if and only if $x = a^y$. The function given by $f(x) = \log_a x$ is called the logarithmic function with base *a*.

all a logarithm is, is an exponent.

Examples: Write in exponential form.

1. $\log_7 343 = 3$ log base 7, exponential base 7, the log is the exponent (log = 3) bad notation!

$7^3 = 343$

3 is the power you raise 7 to in order to get 343

2. $\log_{16} 8 = \frac{3}{4}$

base of log is base of exponential
log is the exponent
to get 8

$16^{3/4} = 8$

3. $\log \frac{1}{1000} = -3$

base unwritten is assumed to be 10
log is exponent
to get $\frac{1}{1000}$

$10^{-3} = \frac{1}{1000}$

4. $\log_8 4 = \frac{2}{3}$

$8^{2/3} = 4$

base is 8
log is 2/3 (exponent)
to get 4

5. You try it: $\log_5 \frac{1}{25} = -2$

Examples: Write in logarithmic form.

1. $13^2 = 169$

$2 = \log_{13} 169$

base of log is base of exponential

log equals exponent

to get 169

2. $9^{3/2} = 27$

base is 9

exponent is 3/2 (log)

to get 27

$\frac{3}{2} = \log_9 27$

3. $4^{-3} = \frac{1}{64}$

base is 4, exponent is -3, to get $\frac{1}{64}$: $-3 = \log_4 \frac{1}{64}$

4. You try it: $10^{-3} = 0.001$

Properties of Logarithms:

1. $\log_a 1 = 0$ because $a^0 = 1$

2. $\log_a a = 1$ because $a^1 = a$

3. $\log_a a^x = x$ and $a^{\log_a x} = x$

4. If $\log_a x = \log_a y$, then $x = y$.

Examples: Simplify

1. $\log_{3.2} 1$

prop 1

$= 0$

anything other than 0 raised to 0 power is 1

2. $9^{\log_9 15}$

prop 3

$= 15$

inverse property

3. $\log_\pi \pi$

prop 2

$= 1$

$\pi^1 = \pi$

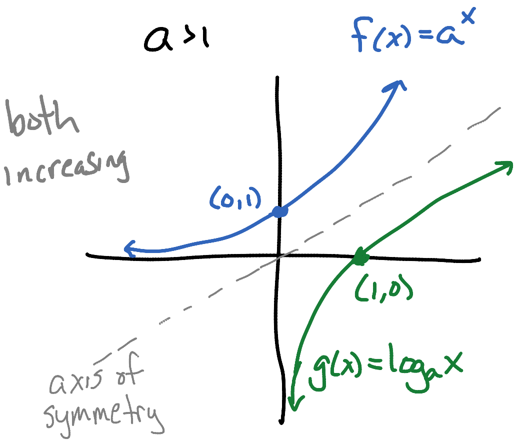
4. $\log_{11} 11^7$

prop 3

$= 7$

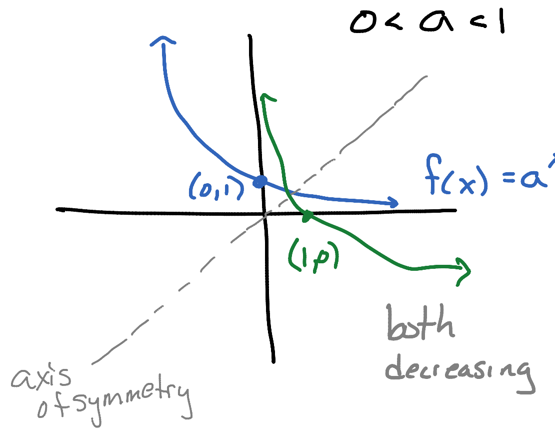
inverse property

Fact: The graph of the logarithmic function is the inverse of the graph of the exponential function. This means that the x-intercept is (1, 0), the domain is $x > 0$ and the range is all real numbers. (Recall the inverse switches x and y, domain and range.) Using this basic knowledge we can move the log graph all over the coordinate system.



Domain $(-\infty,\infty)$
Range $(0,\infty)$

Domain $(0,\infty)$
Range $(-\infty,\infty)$



Definition of the Natural Logarithmic Function – The function defined by $f(x)=\log_e x=\ln x,\ x>0$ is called the natural logarithmic function.

l not I: this is not In(x) it is Lnx, even though lower case l's look like i's to some people, please use common sense!

Fact: All the properties of logarithms still hold with base *e*.

Examples: Use the one-to-one property to solve the equation.

1. $\log_5(x+1)=\log_5 6$

The bases are equal

so $x+1=6$

and $x=5$

2. $\ln(x^2-x)=\ln 6$

The bases are equal

so $x^2-x=6$

$x^2-x-6=0$

$(x-3)(x+2)=0$

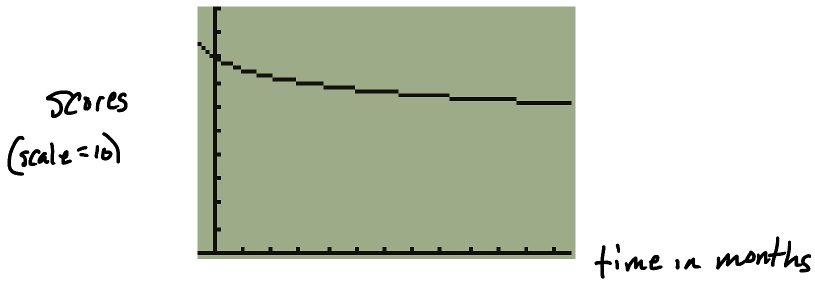
$x-3=0$ or $x+2=0$

$x=3$ or $x=-2$

Domain of a logarithm in general is $x>0$, but we must consider each function separately. Substituting -2 here is just fine.

Example: Students in a mathematics class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model $f(t) = 80 - 17\log(t+1), 0 \le t \le 12$ where t is the time in months.

a) Sketch a basic graph of the function.



b) What was the average score on the original exam (t = 0)?

Solution: The average on the original exam was

$$f(0) = 80 - 17\log(0+1) = 80 - 17\log(1) = 80 - 17(0) = 80$$

c) What was the average score after month 4?

Solution: After month 4 we would use t = 4 to get
$$f(4) = 80 - 17\log(4+1) = 80 - 17\log 5 \approx 68.1$$

d) What was the average score after month 10?

Solution: After month 10 we would use t = 10 to get
$$f(10) = 80 = 17\log(10+1) = 80 - 17\log 11 \approx 62.3$$