3.3 Properties of Logarithms

Change-of-Base Formula – Let *a, b* and *x* be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then $\log_a x$ can be converted to a different base as follows:

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Examples: Rewrite the log as a ratio in two ways.

1. $\log_3 47 = \dfrac{\log 47}{\log 3} = \dfrac{\ln 47}{\ln 3}$

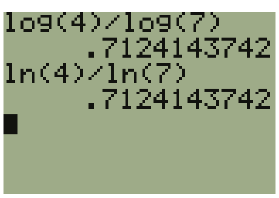
2. $\log_{1/3} x = \dfrac{\log x}{\log \frac{1}{3}} = \dfrac{\ln x}{\ln 1/3}$

3. $\log_x \dfrac{3}{4} = \dfrac{\log 3/4}{\log X} = \dfrac{\ln 3/4}{\ln X}$

The base is already written a little lower (as a subscript) so it seems reasonable that it ends up in the denominator.

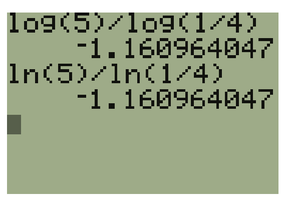
Examples: Evaluate, round to three decimal places.

1. $\log_7 4$



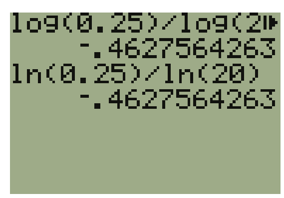
0.712

2. $\log_{1/4} 5$



-1.161

3. $\log_{20} 0.25$



-0.463

It does not matter which one you use. I generally use natural Log because of my calculator.

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Properties of Logarithms – Let 'a' be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, the following properties are true.

1. Product Property - $\log_a(uv) = \log_a u + \log_a v$ or $\ln(uv) = \ln u + \ln v$

2. Quotient Property - $\log_a \frac{u}{v} = \log_a u - \log_a v$ or $\ln \frac{u}{v} = \ln u - \ln v$

3. Power Property - $\log_a u^n = n \log_a u$ or $\ln u^n = n \ln u$

* NO sum or Difference rules exist

Examples: Use the properties of logarithms to expand the expression.

1. $\log_3 10z = \log_3 10 + \log_3 z$

2. $\log 4x^2 y = \log 4 + \log x^2 + \log y$
$= \log 4 + 2\log x + \log y$

3. $\ln \frac{6}{\sqrt{x^2+1}} = \ln \frac{6}{(x^2+1)^{1/2}} = \ln 6 - \ln(x^2+1)^{1/2}$
$= \ln 6 - \frac{1}{2}\ln(x^2+1)$

There are no sum or difference rules. We cannot simplify any further.

Examples: Condense to a single logarithm.

1. $\log_5 8 - \log_5 t = \log_5 \frac{8}{t}$

subtraction becomes division

2. $\log x - 2\log y + 3\log z = \log x - \log y^2 + \log z^3$
$= \log \frac{x}{y^2} + \log z^3$
$= \log \frac{x}{y^2} z^3 = \log \frac{xz^3}{y^2}$

powers first
two at a time to condense after that

3. $\ln x - [\ln(x+1) + \ln(x-1)]$ $= \ln x - [\ln (x+1)(x-1)]$ $= \ln x - \ln(x^2-1)$

parentheses first

then division

$= \ln \frac{x}{(x+1)(x-1)}$ $= \ln \frac{x}{x^2-1}$

either answer is acceptable and completely correct.

4. You try it: $\frac{1}{2}[\log_4(x+1) + 2\log_4(x-1)] + 6\log_4 x$

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