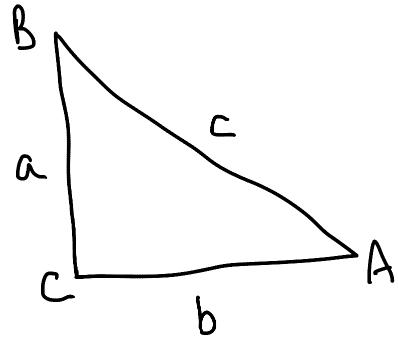
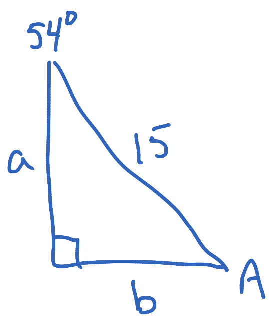
4.8 Applications and Models

Examples: Solve the right triangle with angles A, B, and C with corresponding sides a, b, and c. Round your answers to two decimal places. (C is the right angle.)



1. $B = 54^\circ$, $c = 15$

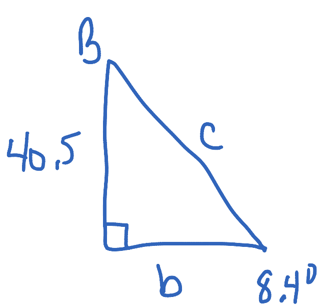


$\sin 54^\circ = \frac{b}{15}$ so $b = 15\sin 54^\circ \approx 12.14 = b$

$\cos 54^\circ = \frac{a}{15}$ so $a = 15\cos 54^\circ \approx 8.82 = a$

$A + B + C = 180^\circ \Rightarrow A + 54^\circ + 90^\circ = 180 \Rightarrow A = 36^\circ$

2. $A = 8.4^\circ$, $a = 40.5$



$A + B + C = 180 \Rightarrow 8.4^\circ + B + 90^\circ = 180^\circ \Rightarrow B = 81.6^\circ$

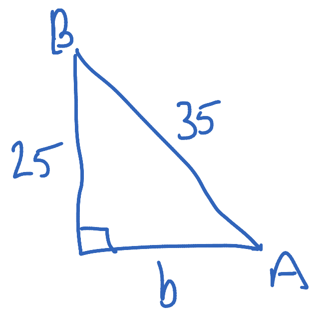
$\tan 8.4^\circ = \frac{40.5}{b}$ so $b\tan 8.4^\circ = 40.5$ and $b = \frac{40.5}{\tan 8.4^\circ}$

≈ 274.27

$\sin 8.4^\circ = \frac{40.5}{c}$

$c\sin 8.4^\circ = 40.5$ so $c = \frac{40.5}{\sin 8.4} = 277.24$

3. $a = 25, c = 35$



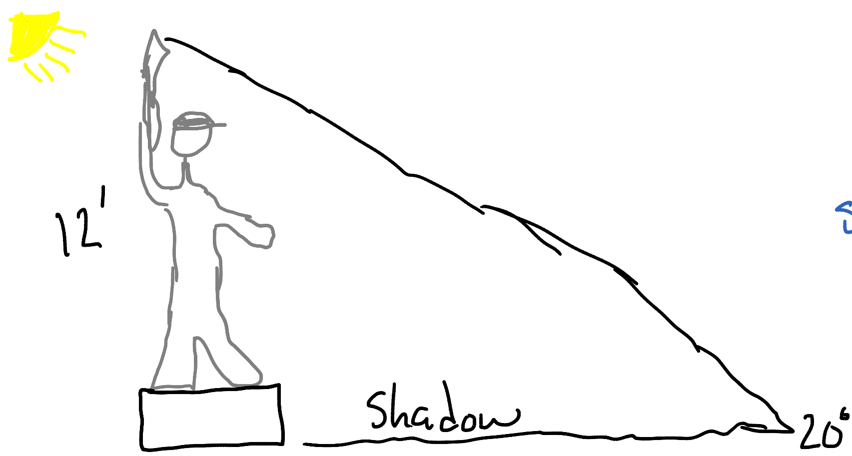
$a^2 + b^2 = c^2$ so $25^2 + b^2 = 35^2 \Rightarrow b^2 = 35^2 - 25^2$

$b^2 = 600 \Rightarrow b \approx 24.49$

$\sin A = \frac{25}{35}$ so $A = \arcsin\left(\frac{25}{35}\right) = 45.58^\circ$

$\cos B = \frac{25}{35}$ so $B = \arccos\left(\frac{25}{35}\right) = 44.42^\circ$

Example: The sun is 20° above the horizon. Find the length of a shadow cast by a park statue that is 12 feet tall.



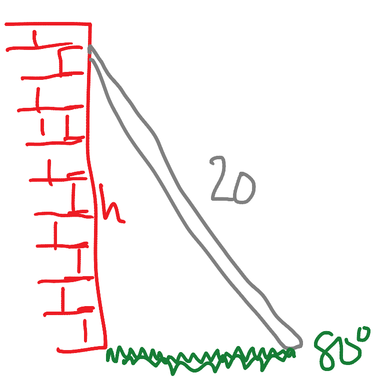
$\tan 20^\circ = \frac{12}{\text{shadow}}$

$\text{shadow} = \frac{12}{\tan 20^\circ}$

$\text{Shadow} = 32.9697$

$\approx 32.97 \text{ ft}$

Example: A ladder 20 feet long leans against the side of a house. Find the height from the top of the ladder to the ground if the angle of elevation of the ladder is 80°.



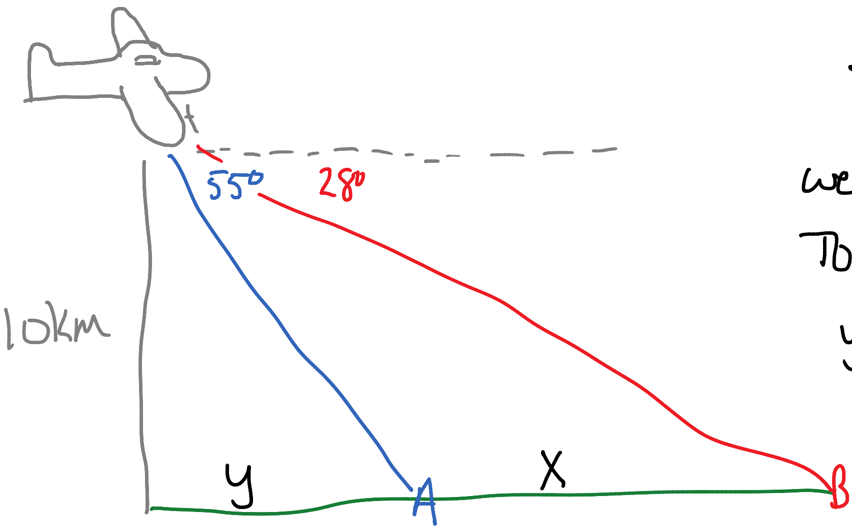
$\sin 80^\circ = \frac{h}{20}$

$20 \sin 80^\circ = h$

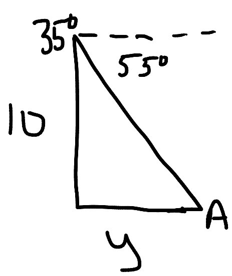
$h = 19.696155$

$h = 19.7 \text{ ft}$

Example: A passenger in an airplane at an altitude of 10 km sees two towns directly to the east of the plane. The angles of depression to the town are 28° and 55°. How far apart are the towns?

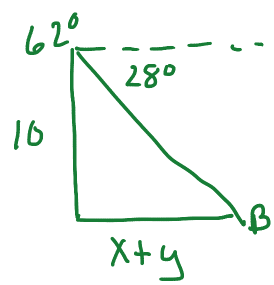


To solve this, we need to find x. To find x, we find y, then find x+y and then subtract to get x



$\tan 35^\circ = \frac{y}{10}$

$y = 7.00$



$\tan 62^\circ = \frac{x+y}{10}$

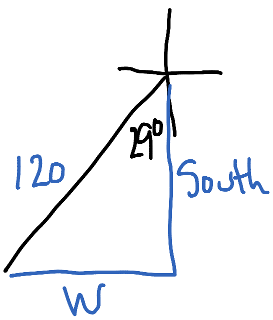
$10\tan 62^\circ = x+y = 18.81$

$x = 18.81 - 7 = 11.81$ km apart

Example: A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots.

(a) How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 PM?

(b) At 6:00 PM, the ship changes course to due west. Find the ship's bearing and distance from the port of departure at 7:00 PM.



a) At 6 pm the ship has sailed

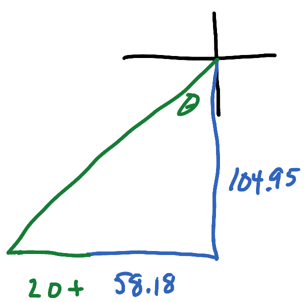
$6(20) = 120$ nm

$\sin 29^\circ = \frac{w}{120}$ so $\text{west} = 120 \sin 29^\circ$

$\text{west} = 58.18$ nm

$\cos 29^\circ = \frac{\text{South}}{120}$ so $\text{south} = 120 \cos 29^\circ$

$\text{south} = 104.95$ nm



$\tan\theta = \frac{20+58.18}{104.95}$

$\theta = \arctan\left(\frac{78.18}{104.95}\right) = 36.7^\circ \leftarrow$ not our final answer!

S 36.7° W

must be in the form of a bearing