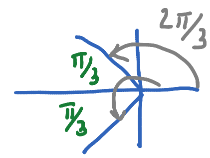
5.3 Solving Trigonometric Equations

Examples: Solve the trigonometric equation.

1. $2\cos x + 1 = 0$



$2\cos x = -1$

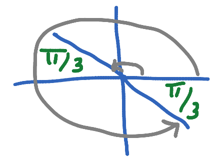
$\cos x = -\frac{1}{2}$

cosine is negative in 2+3

cosine is $\frac{1}{2}$ at $\frac{\pi}{3}$ ref

$X = \frac{2\pi}{3}, \frac{4\pi}{3}$

2. $\tan x + \sqrt{3} = 0$



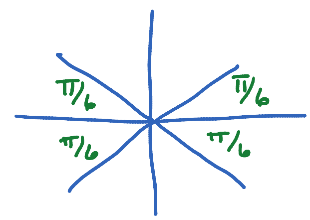
$\tan x = -\sqrt{3}$

tangent is negative in 2+4

tangent is $\sqrt{3}$ when ref angle is $\pi/3$ $\left(\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \cos\frac{\pi}{3} = \frac{1}{2}\right)$

$X = \frac{2\pi}{3}, \frac{5\pi}{3}$

3. $4\sin^2 x - 1 = 0$



$4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\sqrt{\frac{1}{4}}$

$\sin x = \pm\frac{1}{2}$

Sine is positive in 1+2
negative in 3+4

Sine is $\frac{1}{2}$ at $\frac{\pi}{6}$.

$X = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

4. $2\sin^2 x = 2 + \cos x$

$2(1 - \cos^2 x) = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$-2 \quad +2\cos^2 x \quad -2 \quad +2\cos^2 x$

$0 = \cos x + 2\cos^2 x$

$0 = \cos x(1 + 2\cos x)$

$\cos x = 0$

(graph) $X = \frac{\pi}{2}, \frac{3\pi}{2}$

$1 + 2\cos x = 0$

$\cos x = -\frac{1}{2}$ so $X = \frac{2\pi}{3}, \frac{4\pi}{3}$ (previous problem)

5. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$2\sin x + 1 = 0$ or $\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = -1$ from graph

3+4 $\quad \pi/6$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

6. $\cos x + \sin x \tan x = 2$

$\cos x + \sin x \tan x = \cos x + \sin x \frac{\sin x}{\cos x} = \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

Using this identity, we now solve

$\sec x = 2$ but $\sec x = 2$ when $\cos x = \frac{1}{2}$ ← positive in 1+4 ← ref $\pi/3$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

Examples: Use inverse functions to solve.

1. $\tan^2 x - \tan x - 2 = 0$

$(\tan x - 2)(\tan x + 1) = 0$

$\tan x - 2 = 0$ or $\tan x + 1 = 0$

$\tan x = 2$ or $\tan x = -1$

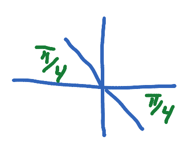
pos in 1+3 $\qquad$ neg in 2+4

tangent is 1 when sine = cosine ref = $\frac{\pi}{4}$

$x = \arctan(2)$ is ref

$x = 63.4°$ in quad 1

$x = 180 + 63.4° = 243.4°$ in quad 2



$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Convert to radians if necessary

2. $2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$2\cos x - 1 = 0$ or $\cos x - 2 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$\cos x = 2$

the range of cosine is $[-1, 1]$ therefore $\cos x \neq 2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

3. $3\tan^2 x + 4\tan x - 4 = 0$

$(3\tan x - 2)(\tan x + 2) = 0$

$3\tan x - 2 = 0$ or $\tan x + 2 = 0$

$3\tan x = 2$

$\tan x = \frac{2}{3}$ positive in 1+3

$\tan x = -2$ negative in 2+4

$x = \arctan(\frac{2}{3}) = 33.69^\circ$

$x = 180 + 33.69 = 213.69^\circ$

$x = \arctan(-2) = -63.4^\circ + 360^\circ = 296.6^\circ$

coterminal angle

$x = 180^\circ - 63.4^\circ = 116.6^\circ$

Example: A Ferris wheel is built such that the height *h* (in feet) above ground of a seat on the wheel at time *t* (in minutes) can be modeled by $h(t) = 53 + 50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$. The wheel makes one revolution every 32 seconds. The ride begins when *t = 0.*

(a) During the first 32 seconds of the ride, when will a person on a Ferris wheel be 53 feet above the ground?

$$53 = 53 + 50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$$
$$-53 \quad -53$$

$$\frac{0}{50} = \frac{50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)}{50}$$

$$0 = \sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$$

we know Sine is zero at $x = 0, \pi, 2\pi$, etc.
This is $x = 0 + n\pi$, where n is an integer.
Now we solve:

$$\frac{\pi}{16}t - \frac{\pi}{2} = 0$$

$$\frac{16}{\pi}\cdot\frac{\pi}{16}t = \frac{\pi}{2}\cdot\frac{16}{\pi}$$

$t = 8$ seconds

$$\frac{\pi}{16}t - \frac{\pi}{2} = \pi$$

$$\frac{\pi}{16}t = \frac{3\pi}{2}$$

$$t = \frac{3\pi}{2}\cdot\frac{16}{\pi}$$

$t = 24$ seconds

A person will be 53 ft above the ground at 8 seconds and 24 seconds.

(b) When will a person be at the top of the Ferris wheel for the first time during the ride? If the ride lasts 160 seconds, how many times will a person be at the top of the ride, and at what times?

$$163 = 53 + 50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$$

$$50 = 50\sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$$

$$1 = \sin\left(\frac{\pi}{16}t - \frac{\pi}{2}\right)$$

Sine is 1 at $\pi/2$:

$$\frac{\pi}{16}t - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\frac{\pi}{16}t = \pi$$

$t = 16$ seconds

top of the wheel

one cycle lasts 32 seconds, so at the top of the ride at 16 sec, 48 sec, 80 sec, 112 sec, 144 seconds and then the ride ends.