

## 6.2 Law of Cosines

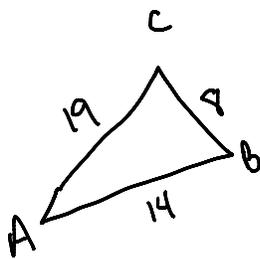
The Law -  $a^2 = b^2 + c^2 - 2bc \cos A$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Don't waste a ton of time memorizing these; notice the Pythagorean nature of these equations and the isolated side is the same as the angle.

Example: A triangle has sides  $a = 8 \text{ ft.}$ ,  $b = 19 \text{ ft.}$ , and  $c = 14 \text{ ft.}$  Find the measure of all three angles.



$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{is} \quad \frac{a^2 - b^2 - c^2}{-2bc} = \cos A$$

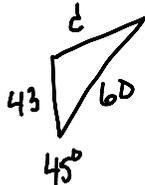
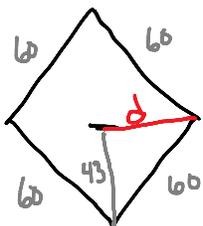
$$22.1^\circ = \cos^{-1}\left(\frac{493}{532}\right) = A \quad \leftarrow \quad \frac{493}{532} = \frac{8^2 - (19)^2 - (14)^2}{-2(19)(14)} = \cos A$$

$$\frac{\sin B}{19} = \frac{\sin 22.1^\circ}{8} \Rightarrow B = \sin^{-1}\left(\frac{19 \sin 22.1^\circ}{8}\right) = 63.3^\circ$$

$$22.1^\circ + 63.3^\circ + C = 180^\circ$$

$$C = 94.6^\circ$$

The pitcher's mound on a women's softball field is 43 feet from home plate and the distance between bases is 60 feet. How far is the pitcher's mound from first base?

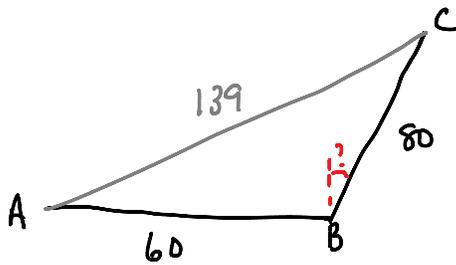


$$d^2 = 43^2 + 60^2 - 2(43)(60) \cos 45^\circ$$

$$d^2 = 1800.329 \dots$$

$$d = 42.43 \text{ ft}$$

A ship travels 60 miles due east, then adjusts its course northward (point B). After traveling 80 miles in that direction, the ship is 139 miles from its point of departure (point C). Describe the bearing from point B to point C.



Bearing from B to C will be the full angle  $B - 90^\circ$ .

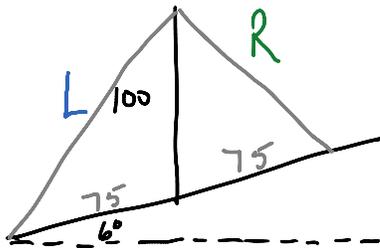
$$\frac{139^2 - 80^2 - 60^2}{-2(80)(60)} = \cos B = \frac{-3107}{3200}$$

$$B = \cos^{-1}\left(\frac{-3107}{3200}\right) = 166.15^\circ$$

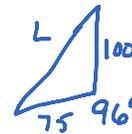
Bearing is  $166.15 - 90 = 76.15^\circ$

**N 76.15° E**

A 100-foot vertical tower is to be erected on the side of a hill that makes a 6 degree angle with the horizontal. Find the length of each of the two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.



The left wire:

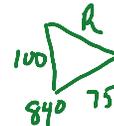


$$L^2 = 100^2 + 75^2 - 2(100)(75)\cos 96^\circ$$

$$L^2 = 17192.926949$$

$$L = 131.1 \text{ ft}$$

The right wire:



$$R^2 = 100^2 + 75^2 - 2(100)(75)\cos 84^\circ$$

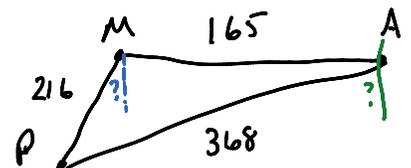
$$R^2 = 14057.073051$$

$$R = 118.6 \text{ ft}$$

On a map, Minneapolis is 165 mm due west of Albany, Phoenix is 216 mm from Minneapolis, and Phoenix is 368 mm from Albany.

→ a) Find the bearing of Minneapolis from Phoenix.

→ b) Find the bearing of Albany from Phoenix.

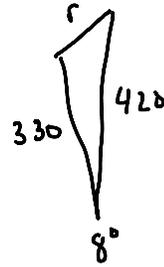
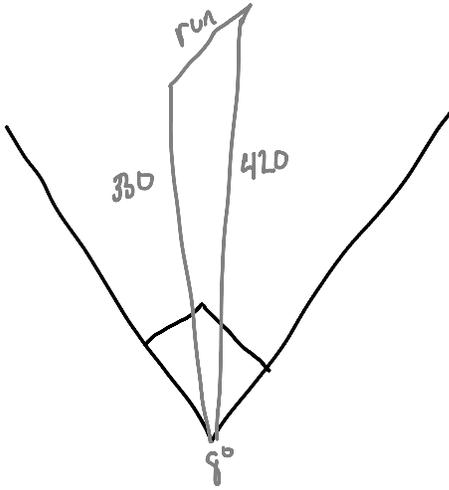


$$\frac{368^2 - 165^2 - 216^2}{-2(165)(216)} = \cos M = \frac{-61543}{71280} \Rightarrow M = 149.7^\circ \text{ Bearing: } S 59.7^\circ W$$

$$\frac{216^2 - 368^2 - 165^2}{-2(368)(165)} = \cos A = \frac{115993}{121440} \Rightarrow A = 17.2^\circ \text{ Bearing: } S 72.8^\circ W$$

$$90 - 17.2^\circ = 72.8^\circ$$

The baseball player in center field is playing approximately 330 feet from the TV camera that is behind home plate. A batter hits a fly ball that goes to the wall 420 feet from the camera. The camera turns 8 degrees to follow the play. Approximately how far does the center fielder have to run to make the catch?



$$r^2 = 330^2 + 420^2 - 2(330)(420)\cos 8^\circ$$

$$r^2 = 10797.6913448$$

$$r = 103.9 \text{ ft}$$