7.2 Two-Variable Linear Systems

Method of Elimination – To use the method of elimination to solve a system of two linear equations in x and y, perform the following steps

1. Obtain coefficients for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.

2. Add the equations to eliminate one variable.

3. Solve the equation obtained in step 2.

4. Back-substitute the value obtained in step 3 into either of the original equations and solve for the other variable.

5. Check that the solution satisfies each of the original equations.

Examples: Solve by elimination.

1. $\begin{cases} 3x - 5y = 8 \\ 2x + 5y = 22 \end{cases}$

① The coefficients of y already differ only in sign

② $3x - 5y = 8$
$2x + 5y = 22$

③ $\dfrac{5x}{5} = \dfrac{30}{5}$

$x = 6$

④ $3(6) - 5y = 8$
$18 - 5y = 8$
$-5y = -10$
$y = 2$

OR

either equation gives (6,2)

④ $2(6) + 5y = 22$
$12 + 5y = 22$
$5y = 10$
$y = 2$

2. $\begin{cases} x + 5y = 10 \\ 3x - 10y = -5 \end{cases}$

① eliminate y:

$2(x + 5y = 10) \\ 1(3x - 10y = -5)$ = $\begin{array}{l} 2x + 10y = 20 \\ 3x - 10y = -5 \end{array}$

$\dfrac{5x = 15}{}$
$x = 3$

$(3) + 5y = 10$
$5y = 7$
$y = \dfrac{7}{5}$

Solution: $(3, \dfrac{7}{5})$

3. $\begin{cases} 3x+11y=4 \\ -2x-5y=9 \end{cases}$ eliminate x or y? x seems "smaller"

$2(3x+11y=4) \Rightarrow \begin{array}{l} 6x+22y=8 \\ -6x-15y=27 \end{array}$ $\Rightarrow 3x+11(5)=4$
$3(-2x-5y=9)$ $\underline{}$ $3x+55=4$
 $7y=35$ $\dfrac{3x}{3}=\dfrac{-51}{3}$
 $y=5$ $x=-17$

Solution:
$(-17,5)$

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4. $\begin{cases} 7x+8y=6 \\ -14x-16y=-12 \end{cases}$ let's eliminate x

$2(7x+8y=6) \Rightarrow \begin{array}{l} 14x+16y=12 \\ -14x-16y=-12 \end{array}$
$1(-14x-16y=-12)$ $\underline{}$
 $0=0$ ← always true is a dependent system.

To find the solution for webAssign, let $y=a$ and solve for x:

$7x+8a=6 \rightarrow 7x=6-8a \rightarrow x=\dfrac{6-8a}{7}$ Solution: $\left(\dfrac{6-8a}{7}, a\right)$

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5. $\begin{cases} 0.05x-0.03y=0.21 \\ 0.07x+0.02y=0.16 \end{cases}$ let's multiply everything by 100 to make this easier to look at.

$5x-3y=21$
$7x+2y=16$
now eliminate a variable... You try it!

Example: An airplane flying into a headwind travels the 1800-mile flying distance between Pittsburgh and Phoenix in 3 hours and 36 minutes. On the return flight, the distance is traveled in 3 hours. Find the airspeed of the plane and the speed of the wind, assuming both remained constant.

Pitt → Phx $D=1800, t=3h 36m = 3+\frac{36}{60}h = 3.6$ hrs
 rate $= p-w$

Phx → Pitt $D=1800, t=3hr,$ rate $= p+w$

$\begin{array}{l} 1800=3.6(p-w) \\ 1800=3(p+w) \end{array} \Rightarrow \begin{array}{l} 500=p-w \\ 600=p+w \end{array}$ $\underline{}$
 divide by $1100=2p$
 3.6 and 3 $550=p$

$500=550-w$
$-50=-w$
$50=w$

first thought is D=rt

let $p=$ speed of plane
$w=$ speed of wind

wind speed = 50 mph
plane speed = 550 mph

Example: One eight-ounce glass of apple juice one eight-ounce glass of orange juice contain a total of 177.4 mg of vitamin C. Two eight-ounce glasses of apple juice and three eight-ounce glasses of orange juice contain a total of 436.7 mg of vitamin C. How much vitamin C is in an eight-ounce glass of each type of juice?

Let A = amount of vit C in apple juice

Let J = amount of vit C in orange juice

$$-2\left(\;1A+1J=177.4\right)$$
$$1\left(2A+3J=436.7\right) \implies$$

$$-2A-2J=-354.8$$
$$2A+3S=436.7$$
$$\overline{\qquad\qquad\qquad}$$
$$J=81.9 \text{ vit C in Orange Juice}$$
$$\text{mg}$$

$$A+81.9=177.4$$
$$A=95.5 \text{ mg vit C in apple juice}$$

Example: A total of $32,000 is invested in two municipal bonds that pay 5.75% and 6.25% simple interest. The investor wants an annual interest income of $1900 from the investments. What amount should be invested in the 5.75% bond? — find x

amount in
Let x = bond at 5.75%
y = bond at 6.25%

$$X+y=32000$$
$$.0575x+.0625y=1900$$

I would use substitution, not elimination.

$$y=32,000-X$$

$$.0575x+.0625(32000-x)=1900$$
$$.0575x+2000-0.0625x=1900$$
$$-0.005\,x=-100$$
$$x=\$20,000 \text{ at } 5.75\%$$