Math 1508

Transformations

For any of our parent functions, we can move them about the coordinate system using both rigid and non-rigid transformations.

**Rigid transformations** keep the original shape and size of the graph but moves the entire graph horizontally, vertically, or is a mirror image.

Given any function f(x), the transformed graph given by $g(x) = f(x - h) + k$ is the graph of the original subject to:

- Horizontal shift *h* units. (On the inside so deals directly with *x* is left/right.)
    - For $g(x) = f(x - h) + k$, this shift is to the right.
    - For $g(x) = f(x + h) + k$, this shift is to the left.
    - Trick: Solve $x - h = 0$ or $x + h = 0$ to find the direction if necessary.
- Vertical shift *k* units. (On the outside so deals directly with *y* is up/down.)
    - For $g(x) = f(x - h) + k$, this shift is up.
    - For $g(x) = f(x - h) - k$, this shift is down.
    - Trick: After you apply the function, you either add or subtract to move your graph up or down.

Given any function f(x), the transformed graph given by

- $g(x) = f(-x)$ is a horizontal reflection across the *y*-axis.
- $g(x) = -f(x)$ is a vertical reflection across the *x*-axis.

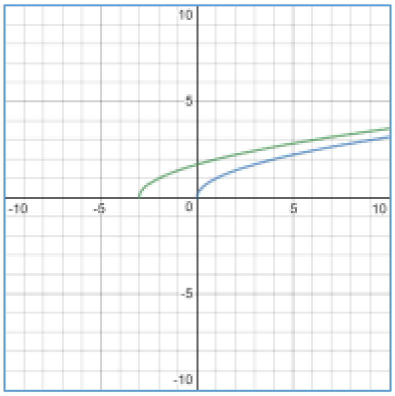
**Non-rigid transformations** stretch of shrink the shape of the graph. It will still have its basic recognizable shape, but may be wider or narrower.

Given any function f(x), the transformed graph given by $g(x) = af(bx)$ is the graph of the original subject to:

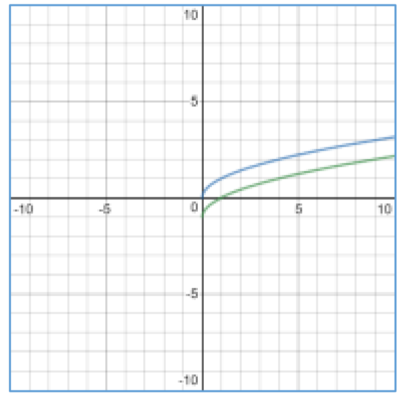
- A vertical stretch of *a* units if $a > 1$ and a vertical shrink of *a* units if $0 < a < 1$. A vertical stretch is like taking the ends of the graph and pulling it upward. This naturally makes the graph thinner. A vertical shrink is like pushing the graph toward the x-axis making the graph wider.
- A horizontal stretch of *b* units if 0<b<1 and a horizontal shrink of b units if b>1. A horizontal stretch is like taking the ends of the graph and pulling out to the sides. This naturally makes the graph wider. A horizontal shrink is like pushing the graph toward the *y*-axis making the graph thinner.
- Notice that the vertical stretch and the horizontal shrink have the same effects on the graph.

Rigid transformations using $f(x) = \sqrt{x}$.

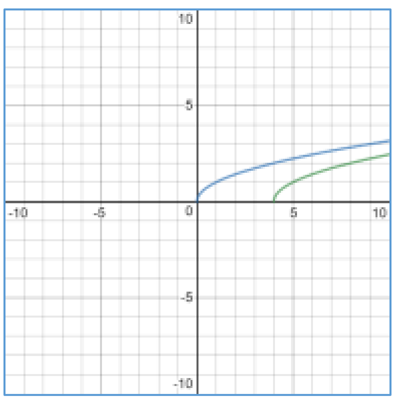
Shift left three units: $y = \sqrt{x+3}$



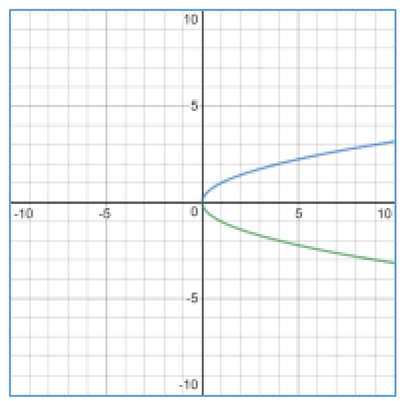

Shift down one unit: $y = \sqrt{x} - 1$



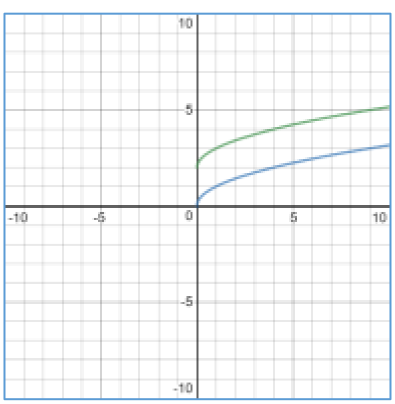

Shift right four units: $y = \sqrt{x-4}$



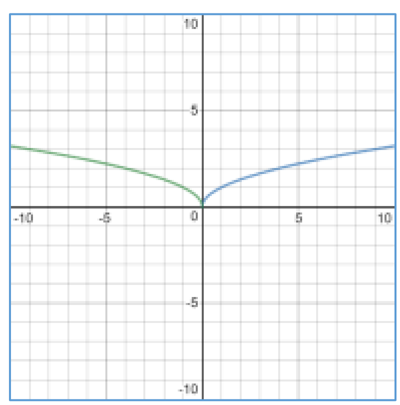

Reflect across the *x*-axis: $y = -\sqrt{x}$




Shift up two units: $y = \sqrt{x} + 2$

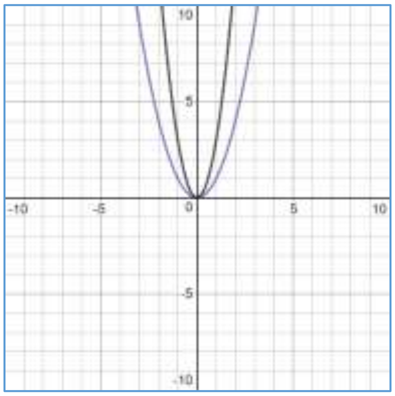



Reflect across the *y*-axis: $y = \sqrt{-x}$

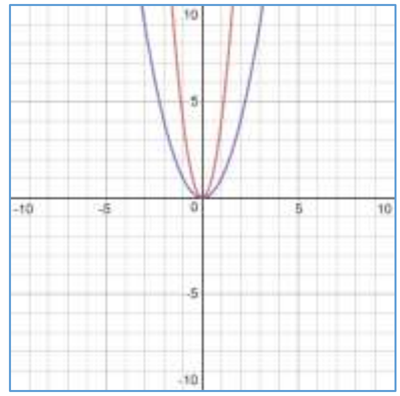


Non-rigid transformations using $f(x) = x^2$

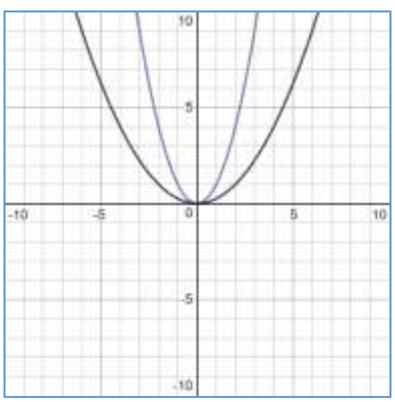
Vertical stretch by a factor of 3: $y = 3x^2$




Horizontal shrink by a factor of 2: $y = (2x)^2$




Vertical shrink by a factor of 4: $y = \frac{1}{4}x^2$




Horizontal stretch by a factor of 3: $y = \left(\frac{1}{5}x\right)^2$