11.2 A First Application: Marginal Analysis

Marginal Cost – A cost function specifies the total cost C as a function of the number of items x. In other words, $C(x)$ is the total cost of x items. The marginal cost function is the derivative, $C'(x)$, of the cost function, $C(x)$. This derivative measures the rate of change of cost with respect to x. The units of marginal cost are the units of cost per item. We interpret $C'(x)$ as the approximate cost of one more item.

Marginal Revenue and Profit – A revenue or profit function specifies the total revenue R or profit P as a function of the number of items x. The derivatives, $R'(x)$ and $P'(x)$, of these functions are called the marginal revenue and marginal profit functions. They measure the rate of change of revenue and profit with respect to x. The units of marginal revenue and profit are the same as those of marginal cost: dollars (or euros, pesos, etc.) per item. We interpret $R'(x)$ and $P'(x)$ as the approximate revenue and profit from the sale of one more item.

Examples

1. The cost of producing x teddy bears per day at the Cuddly Companion Co. is calculated by their marketing staff to be given by the formula $C(x)=100+40x-0.001x^2$.

   a) Find the marginal cost function and use it to estimate how fast the cost is going up at a production level of 100 teddy bears. Compare this with the exact cost of producing the 101st teddy bear.

   $C'(x) = 40 - 0.001(2x)$ or $C'(x) = 40 - 0.002x$

   $C'(100) = 40 - 0.002(100) = 40 - 0.2 = \$39.80$ / bear

   $C(101) = 100 + 40(101) - 0.001(101)^2 = 4129.80$

   $- \; C(100) = 100 + 40(100) - 0.001(100)^2 = 4090.00$

   Cost of 101st bear $\quad \$39.80$ same!

   b) The average cost function, $\bar{C}(x)$, is given by $\bar{C}(x)=\dfrac{C(x)}{x}$. Find the average cost function and evaluate $\bar{C}(100)$. What does this answer tell you?

$$\bar{C}(x) = \frac{100+40x-0.001x^2}{x} = \frac{100}{x} + 40 - 0.001x$$

$$\bar{C}(100) = \frac{100}{100} + 40 - 0.001(100) = 40.90$$

→ This means in making the first 100 bears, it cost the company \$40.90 per bear.

2. The Audubon Society at ESU is planning its annual fundraising "Eat-a-thon." The society will charge students \$1.10 per serving of pasta. The society estimates that the total cost of producing x servings of pasta at the event will be $C(x) = 350 + 0.10x + 0.002x^2$ dollars.

a) Calculate the marginal revenue and profit functions.

Revenue is 1.10 per serving $\Rightarrow R(x) = 1.10x$

marginal revenue is $R'(x) = 1.10$

Profit is revenue − cost $= R(x) - C(x) = 1.10x - (350 + 0.10x + 0.002x^2)$

which simplifies to $P(x) = -0.002x^2 + x - 350$

with marginal profit $P'(x) = -0.004x + 1$

b) Compute the revenue and profit, and also the marginal revenue and profit, if you have produced and sold 200 servings of pasta. Interpret the results.

$R(200) = 1.10(200) = \$220$

$R'(200) = 1.10$

$P(200) = -0.002(200)^2 + 200 - 350 = -\$230$

$P'(200) = -0.004(200) + 1 = 0.20$

After selling 200 servings of pasta, the club has revenue of \$220 with revenue per plate a constant \$1.10/serving. However, those 200 servings has the club still losing \$230 on the deal. The good news is that the profit per serving is \$0.20 and is positive so profit is increasing.

c) For which value of x is the marginal profit zero? Interpret your answer.

$P'(x) = 0$ when $-0.004x + 1 = 0$

$1 = 0.004x$

$250 = \frac{1}{0.004} = x$

At 250 servings the profit per serving is \$0.

3. The cost *C* of building a house is related to the number *k* of carpenters used and the number *x* of electricians used by the formula $C = 15,000 + 50k^2 + 60x^2$.

a) Assuming that 10 carpenters are currently being used, find the cost function *C*, the marginal cost C' and average cost function $\bar{C}$, all as functions of *x*.

$K = 10$

$C(x) = 15,000 + 50(10)^2 + 60x^2 = 15,000 + 5000 + 60x^2$

so $C(x) = 20,000 + 60x^2$

$C'(x) = 120x$ and $\bar{C}(x) = \frac{20,000 + 60x^2}{x} = \frac{20,000}{x} + 60x$

b) Use the functions you obtained to compute $C'(15)$ and $\bar{C}(15)$. Use these two answers to say whether the average cost is increasing or decreasing as the number of electricians increases.

$C'(15) = 120(15) = \$1800/\text{electr.}$ for 10 carpenters + 15 electricians

$\bar{C}(15) = \frac{20,000}{15} + 60(15) = \$2233.33/\text{electrician}$

Average cost is decreasing because the cost per electrician is less than the average cost per electrician at 15 electricians.