

Math 2301 Section 11.5: Derivatives of Logs and Exponential Functions Algebra Supplement

The algebra you need to know for this section includes, but is not limited to, the definition and manipulation of exponential and logarithmic functions. What I will present here is just the basic version of each of these. Please see sections 9.2 and 9.3 for more discussion under the Math 1320 heading.

Definition – A function $f(x) = a(b)^x$ is called an exponential function if $b > 0$ and $b \neq 1$. The point $(0, a)$ is the y-intercept of this function and we call b the base multiplier; this is the amount you multiply by in order to get the next y when x increases by 1.

Fact – Exponential functions are always increasing when $b > 1$, and always decreasing when $0 < b < 1$.

Due to this fact of monotonicity (always increasing or always decreasing), exponential functions have inverses. We can use algebraic manipulation to try to find the inverse but we get stuck along the way. Whenever this situation arises, mathematicians create fixes. In this case, they created the logarithm.

Math Definition – The function $y = \log_b x$ is the power to which you raise b in order to get x .

My Definition – All a logarithm is, is an exponent. It is the specific exponent that you need to raise the base b to in order to get a set value of x .

Correlation equations: $y = \log_b x$ is $b^y = x$

We will be dealing with general logarithms of random base b with the restrictions given in the first definition. But we will be doing more work with the natural exponential and natural logarithmic functions.

Definition – The number e is given by $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$.

Don't worry, we're not getting back into limits here ... yet. This number e is an irrational number (much like π) so it does not have an accurate decimal representation. We say that $e \approx 2.718$, but this is not an exact value. Always use the number e that is found on your calculator and not a decimal representation.

Natural Exponential Function – The function $f(x) = e^x$ is called the natural exponential function.

Natural Logarithmic Function – The function $f(x) = \ln x$ is called the natural logarithmic function.

Note: The Natural Logarithmic function uses the first letter of each of those words to make LN. The function is not an 'l', it is an 'L', for Logarithm. Do not let the type fool you! (Originally it was *logarithmicae naturalis*.)