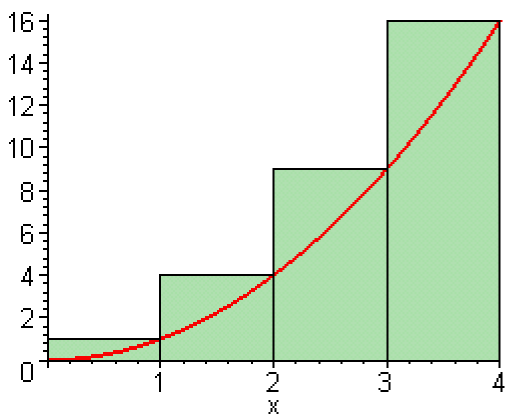
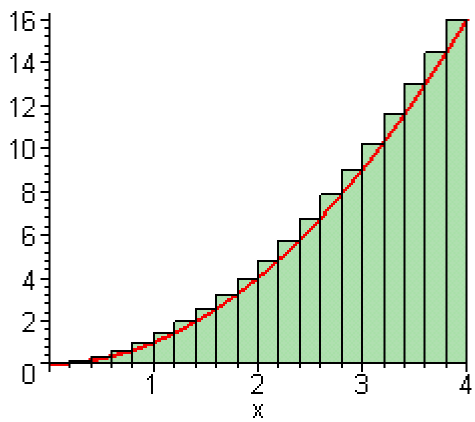
13.3 The Definite Integral: Numerical and Graphical Approaches

This is an important section that shows how anti-derivatives measure area and are infinite sums.

A definite integral involves a function *f(x*) and a closed interval *[a, b*] where we restrict our attention. An indefinite integral would be $\int f(x)\,dx$ but the definite integral is $\int_a^b f(x)\,dx$ and tells us that we no longer need the integration constant *C*. The definite integral measures the area between the curve *f(x*) and the *x*-axis over the interval *[a, b*] so is an actual value.

The numerical approach takes a graph and breaks it into rectangles. We use rectangles because the area is easy to find. We can use many different widths of rectangles to approximate this integral.





In these two graphs of the same function, which one do you think would be closer to the actual area of the region under the red curve and above the x-axis?

Calculus allows us to make our rectangle widths arbitrarily small; this is where the dx comes from. The way it is done is with the use of limits. We won't be doing any homework problems from this section, but the basic idea is something that I want you to at least be exposed to.