INTRO

Welcome to Biographies in Mathematics brought to you from the campus of the University of Texas El Paso by students in my history of math class. My name is Tuesday Johnson and I'll be your host on this tour of time and place to meet the people behind the math.

EPISODE 1: EMMY NOETHER

There were two women I first learned of when I started college in the fall of 1990: Hypatia of Alexandria and Emmy Noether. While Hypatia will be the topic of another episode, it was never a question that I would have Amalie Emmy Noether as my first topic of this podcast.

Emmy was born in Erlangen, Germany, on March 23, 1882 to Max and Ida Noether. Her father Max came from a family of wholesale hardware dealers, a business his grandfather started in Bruchsal (brushal) in the early 1800s, and became known as a great mathematician studying algebraic geometry. Max was a professor of mathematics at the University of Erlangen as well as the Mathematics Institute in Erlangen (MIE). Her mother, Ida, was from the wealthy Kaufmann family of Cologne. Both of Emmy’s parents were Jewish, therefore, she too was Jewish. (Judiasm being passed down a matrilineal line.) Though Noether is not a traditional Jewish name, as I am told, it was taken by Elias Samuel, Max’s paternal grandfather when in 1809 the State of Baden made the Tolerance Edict, which required Jews to adopt Germanic names.

Emmy was the oldest of four children and one of only two who survived childhood. Her brother Fritz also made a career out of mathematics. As a typical girl growing up in late 19th century Germany, she was expected to help with cooking, cleaning, and the other “womanly” duties of the household. Later in life she admitted that she had little patience nor aptitude for those tasks. (I fall into the same tendencies, just ask either my college roommate or partner.)
While her mother loved to play piano, Emmy did not enjoy the lessons forced upon her but she did like to dance as her mother played. She claimed that as a youngster, this, the dancing, was her passion.

As a child, Emmy did not concentrate on math, but spent her time studying languages, with a concentration in French and English. At the time of her graduation from high school, she passed a test that allowed her to teach both subjects at any school for young women.

Noether never became a language teacher however. Instead, she decided to take the difficult route for a woman of that time and study mathematics at university. Women were allowed to study at German universities unofficially and each professor had to give permission for his course. Noether obtained permission to sit in on courses at the University of Erlangen from 1900 to 1902. She was one of only two female students sitting in on courses at Erlangen and, in addition to mathematics courses; she continued her interest in languages being taught by the professor of Roman Studies and by an historian. At the same time, she was preparing to take the examinations which allowed a student to enter any university. Having taken and passed this matriculation examination in Nuremberg on 14 July 1903, she went to the University of Göttingen (gert-in-en).

During the years 1903 and 1904, she attended lectures by Karl Schwarzschild, Otto Blumenthal, David Hilbert, Felix Klein and Hermann Minkowski. Again, she was not allowed to be a properly matriculated student but was only allowed to sit in on lectures. After one semester at Göttingen she returned to Erlangen. That one semester was enough for Emmy to make an impression though.

At this point the rules were changed and women students were allowed to matriculate on an equal basis with the men. On 24 October 1904, Noether enrolled at Erlangen where she now
focused on mathematics. In 1907, she was granted a doctorate after working under Paul Gordan. The oral examination took place on Friday 13 December and she was awarded the degree 'summa cum laude'. Hilbert's basis theorem of 1888 had given an existence result for finiteness of invariants in $n$ variables. Gordan, however, took a constructive approach and looked at constructive methods to arrive at the same results. Noether's doctoral thesis followed this constructive approach of Gordan and listed systems of 331 covariant forms.

Abstract algebra is where Emmy made the most contributions, which landed her in my studies, and she is commonly referred to as a central figure in the field. Much of this is due to her biggest influence, Ernst Fischer. Fischer became the successor of Paul Gordan after he retired. In fact, Emmy is quoted as saying, “Above all I am indebted to Mr. E Fischer from whom I received the decisive impulse to study abstract algebra from an arithmetical viewpoint, and this remained the governing idea for all my later work.”

Now that Emmy Noether had her doctorate in mathematics, she was ready to find a job teaching. The University of Erlangen would not hire her, as they had a policy against women professors. She decided to help her father at the Mathematics Institute in Erlangen. She began doing research there, and helped her father by teaching his classes when he was sick. Soon, she began to publish papers on her own work.

During the years Emmy worked with her father, Germany became involved in World War I. Emmy was a pacifist at heart, and hated the war. In 1918 the war ended, the German monarchy was removed, and the country became a republic. Noether, and all women in Germany, were given the right to vote for the first time. Even with the new rights granted to women, Noether was not paid for her work teaching. The entire time Emmy taught at the
Mathematical Institute of Erlangen, she did so without pay. Even though she had to endure the next 7 years of her life at the institute without a check or a title, she was able to focus most of her time in the realm of theoretical algebra, which would lay the foundation for some of the work she is most importantly known for today.

In 1915, Felix Klein and David Hilbert were working on further defining one of Einstein's theories at the University of Gottingen. They felt that Emmy Noether's expertise could help them in their work. The reason for this was that Hilbert was working on physics, in particular on ideas on the theory of relativity close to those of Albert Einstein. He decided that he needed the help of an expert on invariant theory and, after discussions with Klein, they issued the invitation. Bartel Leender Van der Waerden, a Dutch mathematician and colleague of Heisenenberg at Leipzig, writes [68]:-

*She came and at once solved two important problems. First: How can one obtain all differential covariants of any vector or tensor field in a Riemannian space? ... The second problem Emmy investigated was a problem from special relativity. She proved: To every infinitesimal transformation of the Lorentz group, there corresponds a Conservation Theorem.*

This result in theoretical physics is sometimes referred to as Noether's Theorem, and proves a relationship between symmetries in physics and conservation principles. Einstein praised this basic result in the theory of relativity in a letter to Hilbert when he referred to Noether's penetrating mathematical thinking. “*In the judgment of the most competent living mathematicians, Fräulein Noether was the most significant creative mathematical genius thus far produced since the higher education of women began. In the realm of algebra, in which the
most gifted mathematicians have been busy for centuries, she discovered methods which have proved of enormous importance in the development of the present-day younger generation of mathematicians.”

Hilbert and Klein asked Emmy to come and join them permanently, but since there were no women on the faculty, Noether was unsure if she would be welcome. Many of the faculty did not want her there, but in the end, she came. She worked hard and soon was given a job as a lecturer. Even though she still was not paid for her efforts, for the first time, Noether was teaching under her own name. Three years later, she began receiving a small salary for her work.

Teaching at the University of Gottingen math department until 1933, she went on to advance the world of abstract algebra in various ways and gave birth to doctoral students of her own, some of which were known as the “Noether boys.” Some of these include names such as Chiungtze Tsen, Ernst Witt, Hans Fitting, and Max Deuring. As far as her personal life, she never married and never had children. She spent most of her time devoted to her work and her students.

From 1927 onwards, Noether collaborated with Helmut Hasse and Richard Brauer in work on non-commutative algebras. They wrote a joint paper Beweis eines Hauptsatzes in der Theorie der Algebren (Proof of a main clause in the theory of algebras) which was published in 1932. In addition to teaching and research, Noether helped edit Mathematische Annalen. Much of her work appears in papers written by colleagues and students, rather than under her own name. Further recognition of her outstanding mathematical contributions came with invitations to address the International Congress of Mathematicians at Bologna in September 1928 and again at Zürich in September 1932. Her address to the 1932 Congress was entitled Hyperkomplexe Systeme in ihren
Beziehungen zur kommutativen Algebra und zur Zahlentheorie (Hypercomplex systems in their relations to commutative algebra and number theory). In 1932, she also received, jointly with Emil Artin, the Alfred Ackermann-Teubner Memorial Prize for the Advancement of Mathematical Knowledge.

In mathematics, she focused most of her time specifically in the realms of abstract algebra and topology. In fact, her theorem on continuous symmetry has proved to be very useful in physics and researchers are still finding applications of it today, such as in quantum field theory and the calculation of entropy in stationary black holes. All though all of this is impressive, she suffered greatly from a divided Germany and from the traditional views of women in science.

Einstein writes, “Her unselfish, significant work over a period of many years was rewarded by the new rulers of Germany with a dismissal, which cost her the means of maintaining her simple life and the opportunity to carry on her mathematical studies.” — A.E.

As Noether made her career in what went on to become Nazi Germany, she was posed with many problems in her later years. When Hitler came to power in 1933, he passed what was called the “Law for the Restoration of the Professional Civil Service,” which demanded that members of the Jewish community could not continue their profession unless they exhibited loyalty to Germany by having fought in World War I. Thus Noether, a Jewish woman, was forced to resign her position at the University of Gottingen.

At this time, Emmy accepted a one-year visiting professorship at Bryn Mawr College in the US and in October 1933 sailed to the United States on the ship Bremen to take up the appointment. She had hoped to delay accepting the invitation since she would have liked to go to
Oxford in England but it soon became clear that she had to leave quickly. At Bryn Mawr, Anna Johnson Pell Wheeler, head of mathematics, made her very welcome. Noether ran a seminar during the winter semester of 1933-34 for three students and one member of the staff. They worked through the first volume of van der Waerden's *Moderne Algebra*. In February 1934, she began giving weekly lectures at the Institute for Advanced Study at Princeton. In a letter to Hasse, dated 6 March 1934, she wrote:

*I have started with representation modules, groups with operators ...; Princeton will receive its first algebraic treatment this winter, and a thorough one at that. My audience consists mostly of research fellows, besides Albert [Einstein] and [Harry] Vandiver, but I'm beginning to realise that I must be careful; after all, they are essentially used to explicit computation and I have already driven a few of them away with my approach.*

Noether returned to Germany in the summer of 1934. There see saw her brother Fritz for what would be the last time. She also visited Artin in Hamburg before going on to Göttingen. In 1980 Artin's wife recalled Noether's visit:

*Now the one thing I remember most vividly is the trip on the Hamburg Untergrund, which is the subway in Hamburg. We picked up Emmy at the Institute, and she and Artin immediately started talking mathematics. At that time it was Idealtheorie, and they started talking about Ideal, Führer, and Gruppe, and Untergruppe, and the whole car suddenly started pricking up their ears. [Each of the German nouns has both mathematical and political meanings.] And I was frightened to death - I thought, my*
goodness, next thing's going to happen, somebody's going to arrest us. Of course, that was in 1934, and all. But Emmy was completely oblivious, and she talked very loudly and very excitedly, and got louder and louder, and all the time the "Führer" came out, and the "Ideal." She was very full of life, and she constantly talked very fast and very loud.

She returned to the United States where her visiting professorship at Bryn Mawr had been extended for a further year. She continued her weekly lectures at Princeton where Richard Brauer had now arrived. After her lectures, she enjoyed talking about mathematics with Weyl, Veblen and Brauer.

Einstein called her two years at Pennsylvania’s Bryn Mawr “the happiest and perhaps the most fruitful of her entire career.”

Noether was a voracious researcher/author and produced over 40 papers. It is in some of these publications that she went on to prove Galois theory, Noether’s theorems, the equivalence of the ascending chain condition, and the minimal set of conditions for prime ideals. The most important of these, and, as mentioned earlier, her claim to fame, is a theorem that carries her name as is known as “Noether’s theorem.” Noether’s theorem uses the concept of symmetry to explain the conservation of physical properties in a system. In other words, every symmetry of an object has a conservation law. For example, in a dramatic fashion, if you punch someone in the face, it would have the same effect anywhere else. Given that, you hit them in the exact same place, with the same exact force.

Noether's death was sudden and unexpected. In April 1935, doctors discovered that she had a tumor. Two days later they operated, finding further tumors, which they believed to be benign
and did not remove. The operation seemed a success and for three days, her condition improved. However, on the fourth day she suddenly collapsed and developed a very high temperature. She died later that day, April 14, 1935.

Emmy Noether is remembered as one of the greatest mathematicians to ever have lived and her influence is engraved in mathematics, and my memory, forever.

I hope you enjoyed this first episode of Biographies in Mathematics. All research and resources for this episode were provided by Ulises Cosio of my History of Mathematics course here at UTEP. You picked a good one Ulises, thank you for your effort.
REFERENCES


