Section 3.8 Newton’s Method:

A technique for approximating the real zeros of a Function.
What is the derivative of a function?
What is the derivative of a function?

\[ f'(x_i) = m \] of the tangent line at some \( x_i \).
Using the point slope formula:

\[ y - y_1 = m(x - x_1) \]

\[ y - y_1 = f'(x_1)(x - x_1) \]
Using the point slope formula:

\[ y - y_1 = m(x - x_1) \]

\[ y - y_1 = f'(x_1)(x - x_1) \]

\[ y - f(x_1) = f'(x_1)(x - x_1) \]
Using the point slope formula:

\[ y - y_1 = m(x - x_1) \]

\[ y - y_1 = f'(x_1)(x - x_1) \]

\[ y - f(x_1) = f'(x_1)(x - x_1) \]

\[ y = f(x_1) + f'(x_1)(x - x_1) \]
Let $y = 0$, since we are looking for the zeros of the function.

$$0 = f(x_1) + f'(x_1)(x - x_1)$$
Let \( y = 0 \), since we are looking for the zeros of the function.

\[
o = f(x_1) + f'(x_1)(x - x_1)
\]

Distribute \( f'(x_1) \)

\[
o = f(x_1) + f'(x_1)x - f'(x_1)x_1
\]
Let $y = o$, since we are looking for the zeros of the function.

$$o = f(x_i) + f'(x_i)(x - x_i)$$

$$o = f(x_i) + f'(x_i)x - f'(x_i)x_i$$

Isolate $f'(x_i)x$

$$f'(x_i)x = f'(x_i)x_i - f(x_i)$$
Let $y = 0$, since we are looking for the zeros of the function.

$$o = f(x_i) + f'(x_i)(x - x_i)$$

$$o = f(x_i) + f'(x_i)x - f'(x_i)x_i$$

$$f'(x_i)x = f'(x_i)x_i - f(x_i)$$

Solve for $x$

$$x = \left[ f'(x_i)/f'(x_i) \right] x_i - \left[ f(x_i)/f'(x_i) \right]$$
Let $y = 0$, since we are looking for the zeros of the function.

\[ o = f(x_i) + f'(x_i)(x - x_i) \]

\[ o = f(x_i) + f'(x_i)x - f'(x_i)x_i \]

\[ f'(x_i)x = f'(x_i)x_i - f(x_i) \]

\[ x = \frac{f'(x_i)}{f'(x_i)}x_i - \frac{f(x_i)}{f'(x_i)} \]

\[ x = x_i - \frac{f(x_i)}{f'(x_i)} \]
• If we do this again and again we have the process which is called Newton’s Method.
Newton’s Method for Approximating the Zeros of a Function:

Let \( f(c) = 0 \), where \( f \) is differentiable on an open interval containing \( c \). Then, to approximate \( c \), use the following steps.

1. Make an initial estimate that is close to \( c \). (A graph is helpful)
2. Determine a new approximation
   \[ x_{n+1} = x_n - f(x_n)/f'(x_n) \]
3. If \( |x_n - x_{n+1}| \) is within the desired accuracy, let \( x_{n+1} \) serve as the final approximation. Otherwise, return to Step 2 and calculate a new approximation.

Each successive application of this procedure is called an iteration.
Graph of $f(x) = 3(x-1)^{1/2} - x$
Newton’s Method: \( f(x) = 3(x-1)^{1/2} - x \)

<table>
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<tr>
<th>Problem 7</th>
<th>( x_n )</th>
<th>( f(x_n) )</th>
<th>( f'(x_n) )</th>
<th>( f(x_n)/f'(x_n) )</th>
<th>( x_n-f(x_n)/f'(x_n) )</th>
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Graph of $f(x) = x^3 + 3$
Newton’s Method: \( f(x) = x^3 + 3 \)

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