Answers to Selected Exercises

Chapter 0

Section 0.1

1. -48 **3.** $2/3$ **5.** -1 **7.** 9 **9.** 1 **11.** 33 **13.** 14 **15.** $5/18$ **17.** 13.31 **19.** 6 **21.** $43/16$ **23.** 0 **25.** `3*(2-5)` **27.** `3/(2-5)` **29.** `(3-1)/(8+6)` **31.** `3-(4+7)/8` **33.** `2/(3+x)-x*y^2` **35.** `3.1x^3-4x^(-2)-60/(x^2-1)` **37.** `(2/3)/5` **39.** `3^(4-5)*6` **41.** `3*(1+4/100)^(-3)` **43.** `3^(2*x-1)+4^x-1` **45.** `2^(2x^2-x+1)` **47.** `4*e^(-2*x)/(2-3e^(-2*x))` or `4*(e^(-2*x))/(2-3e^(-2*x))` **49.** `3(1-(-1/2)^2)^2+1`

Section 0.2

1. 27 **3.** -36 **5.** $4/9$ **7.** $-1/8$ **9.** 16 **11.** 2 **13.** 32 **15.** 2 **17.** x^5 **19.** $-\dfrac{y}{x}$ **21.** $\dfrac{1}{x}$ **23.** x^3y **25.** $\dfrac{z^4}{y^3}$ **27.** $\dfrac{x^6}{y^6}$ **29.** $\dfrac{x^4y^6}{z^4}$ **31.** $\dfrac{3}{x^4}$ **33.** $\dfrac{3}{4x^{2/3}}$ **35.** $1-0.3x^2-\dfrac{6}{5x}$ **37.** 2 **39.** $1/2$ **41.** $4/3$ **43.** $2/5$ **45.** 7 **47.** 5 **49.** -2.668 **51.** $3/2$ **53.** 2 **55.** 2 **57.** ab **59.** $x+9$ **61.** $x\sqrt[3]{a^3+b^3}$ **63.** $\dfrac{2y}{\sqrt{x}}$ **65.** $3^{1/2}$ **67.** $x^{3/2}$ **69.** $(xy^2)^{1/3}$ **71.** $x^{3/2}$ **73.** $\dfrac{3}{5}x^{-2}$ **75.** $\dfrac{3}{2}x^{-1.2}-\dfrac{1}{3}x^{-2.1}$ **77.** $\dfrac{2}{3}x-\dfrac{1}{2}x^{0.1}+\dfrac{4}{3}x^{-1.1}$ **79.** $(x^2+1)^{-3}-\dfrac{3}{4}(x^2+1)^{-1/3}$ **81.** $\sqrt[3]{2^2}$ **83.** $\sqrt[3]{x^4}$ **85.** $\sqrt[5]{\sqrt{x}\sqrt[3]{y}}$ **87.** $-\dfrac{3}{2\sqrt[4]{x}}$ **89.** $\dfrac{0.2}{\sqrt[3]{x^2}}+\dfrac{3\sqrt{x}}{7}$ **91.** $\dfrac{3}{4\sqrt{(1-x)^5}}$ **93.** 64 **95.** $\sqrt{3}$ **97.** $1/x$ **99.** xy **101.** $\left(\dfrac{y}{x}\right)^{1/3}$ **103.** ±4 **105.** $\pm2/3$ **107.** $-1, -1/3$ **109.** -2 **111.** 16 **113.** ±1 **115.** $33/8$

Section 0.3

1. $4x^2+6x$ **3.** $2xy-y^2$ **5.** x^2-2x-3 **7.** $2y^2+13y+15$ **9.** $4x^2-12x+9$ **11.** x^2+2+1/x^2 **13.** $4x^2-9$ **15.** y^2-1/y^2 **17.** $2x^3+6x^2+2x-4$ **19.** $x^4-4x^3+6x^2-4x+1$ **21.** $y^5+4y^4+4y^3-y$ **23.** $(x+1)(2x+5)$ **25.** $(x^2+1)^5(x+3)^3(x^2+x+4)$ **27.** $-x^3(x^3+1)\sqrt{x+1}$ **29.** $(x+2)\sqrt{(x+1)^3}$ **31. a.** $x(2+3x)$ **b.** $x=0, -2/3$ **33. a.** $2x^2(3x-1)$ **b.** $x=0, 1/3$ **35. a.** $(x-1)(x-7)$ **b.** $x=1, 7$ **37. a.** $(x-3)(x+4)$ **b.** $x=3, -4$ **39. a.** $(2x+1)(x-2)$ **b.** $x=-1/2, 2$ **41. a.** $(2x+3)(3x+2)$ **b.** $x=-3/2, -2/3$ **43. a.** $(3x-2)(4x+3)$ **b.** $x=2/3, -3/4$ **45. a.** $(x+2y)^2$ **b.** $x=-2y$ **47. a.** $(x^2-1)(x^2-4)$ **b.** $x=\pm1, \pm2$

Section 0.4

1. $\dfrac{2x^2-7x-4}{x^2-1}$ **3.** $\dfrac{3x^2-2x+5}{x^2-1}$ **5.** $\dfrac{x^2-x+1}{x+1}$ **7.** $\dfrac{x^2-1}{x}$ **9.** $\dfrac{2x-3}{x^2y}$ **11.** $\dfrac{(x+1)^2}{(x+2)^4}$ **13.** $\dfrac{-1}{\sqrt{(x^2+1)^3}}$ **15.** $\dfrac{-(2x+y)}{x^2(x+y)^2}$

Section 0.5

1. -1 **3.** 5 **5.** $13/4$ **7.** $43/7$ **9.** -1 **11.** $(c-b)/a$ **13.** $x=-4, 1/2$ **15.** No solutions **17.** $\pm\sqrt{\dfrac{5}{2}}$ **19.** -1 **21.** $-1, 3$ **23.** $\dfrac{1\pm\sqrt{5}}{2}$ **25.** 1 **27.** $\pm1, \pm3$ **29.** $\pm\sqrt{\dfrac{-1\pm\sqrt{5}}{2}}$ **31.** $-1, -2, -3$ **33.** -3 **35.** 1 **37.** -2 **39.** $1, \pm\sqrt{5}$ **41.** $\pm1, \pm\dfrac{1}{\sqrt{2}}$ **43.** $-2, -1, 2, 3$

Section 0.6

1. $0, 3$ **3.** $\pm\sqrt{2}$ **5.** $-1, -5/2$ **7.** -3 **9.** $0, -1, 1$ **11.** $x=-1$ ($x=-2$ is not a solution.) **13.** $-2, -3/2, -1$ **15.** -1 **17.** $\pm\sqrt[4]{2}$ **19.** ±1 **21.** ±3 **23.** $2/3$ **25.** $-4, -1/4$

Chapter 1

Section 1.1

1. a. 2 **b.** 0.5 **3. a.** -1.5 **b.** 8 **c.** -8 **5. a.** -7 **b.** -3 **c.** 1 **d.** $4y-3$ **e.** $4(a+b)-3$ **7. a.** 3 **b.** 6 **c.** 2 **d.** 6 **e.** a^2+2a+3 **f.** $(x+h)^2+2(x+h)+3$ **9. a.** 2 **b.** 0 **c.** $65/4$ **d.** x^2+1/x **e.** $(s+h)^2+1/(s+h)$ **f.** $(s+h)^2+1/(s+h)-(s^2+1/s)$ **11. a.** 1 **b.** 1 **c.** 0 **d.** 27 **13. a.** Yes; $f(4)=63/16$ **b.** Not defined **c.** Not defined **15. a.** Not defined **b.** Not defined **c.** Yes, $f(-10)=0$ **17. a.** $h(2x+h)$ **b.** $2x+h$ **19. a.** $-h(2x+h)$ **b.** $-(2x+h)$ **21.** `0.1*x^2-4*x+5`

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	1.1	-2.6	-6.1	-9.4	-12.5	-15.4	-18.1	-20.6	-22.9	-25

23. `(x^2-1)/(x^2+1)`

x	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5
$h(x)$	-0.6000	0.3846	0.7241	0.8491	0.9059	0.9360	0.9538	0.9651	0.9727	0.9781	0.9820

25. a. $P(5)=117$, $P(10)=132$, and $P(9.5)\approx131$. Approximately 117 million people were employed in the U.S. on July 1, 1995, 132 million people on July 1, 2000, and 131 million people

on January 1, 2000. **b.** [5, 11]. **27. a.** [0, 10]. $t \geq 0$ is not an appropriate domain because it would predict U.S. trade with China into the indefinite future with no basis. **b.** \$280 billion; U.S. trade with China in 2004 was valued at approximately \$280 billion. **29. a.** (2) **b.** \$36.8 billion **31. a.** 358,600 **b.** 361,200 **c.** \$6.00 **33. a.** $P(0) = 200$: At the start of 1995, the processor speed was 200 megahertz. $P(4) = 500$: At the start of 1999, the processor speed was 500 megahertz. $P(5) = 1100$: At the start of 2000, the processor speed was 1100 megahertz. **b.** Midway through 2001
c:

t	0	1	2	3	4	5	6	7	8	9
P(*t*)	200	275	350	425	500	1100	1700	2300	2900	3500

35. a. `(0.08*t+0.6)*(t<8)+(0.355*t-1.6)*(t>=8)`
b:

t	0	1	2	3	4	5	6	7	8	9	10	11
C(*t*)	0.6	0.68	0.76	0.84	0.92	1	1.08	1.16	1.24	1.595	1.95	2.305

37. $T(26{,}000) = \$730 + 0.15(26{,}000 - 7300) = \$3535$; $T(65{,}000) = \$4090 + 0.25(65{,}000 - 29{,}700) = \$12{,}915$
39. a. \$12,000 **b.** $N(q) = 2000 + 100q^2 - 500q$; $N(20) = \$32{,}000$ **41. a.** `100*(1-12200/t^4.48)`
b:

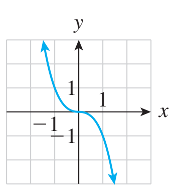
t	9	10	11	12	13	14	15	16	17	18	19	20
p(*t*)	35.2	59.6	73.6	82.2	87.5	91.1	93.4	95.1	96.3	97.1	97.7	98.2

c. 82.2% **d.** 14 months **43.** *t*; *m* **45.** $y(x) = 4x^2 - 2$ (or $f(x) = 4x^2 - 2$) **47.** $N(t) = 200 + 10t$ (N = number of sound files, t = time in days) **49.** As the text reminds us: to evaluate f of a quantity (such as $x + h$) replace x everywhere by the *whole quantity* $x + h$, getting $f(x + h) = (x + h)^2 - 1$. **51.** False: Functions with infinitely many points in their domain (such as $f(x) = x^2$) cannot be specified numerically.

Section 1.2

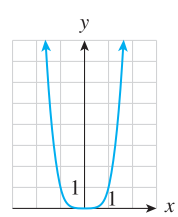
1. a. 20 **b.** 30 **c.** 30 **d.** 20 **e.** 0 **3. a.** −1 **b.** 1.25 **c.** 0 **d.** 1 **e.** 0 **5. a.** (I) **b.** (IV) **c.** (V) **d.** (VI) **e.** (III) **f.** (II)

7.



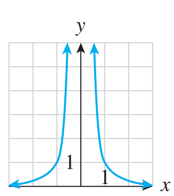
`-(x^3)`

9.



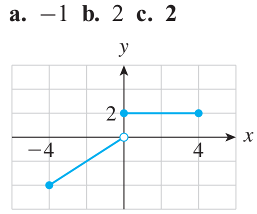
`x^4`

11.



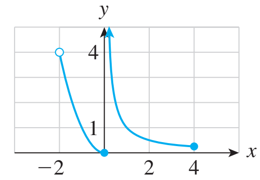
`1/x^2`

13. a. −1 **b.** 2 **c.** 2



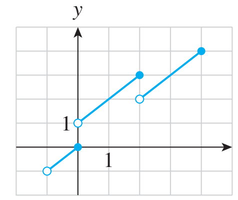
`x*(x<0)+2*(x>=0)`

15. a. 1 **b.** 0 **c.** 1



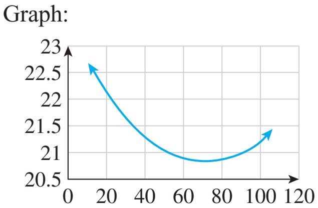
`(x^2)*(x<=0)+(1/x)*(0<x)`

17. a. 0 **b.** 2 **c.** 3 **d.** 3

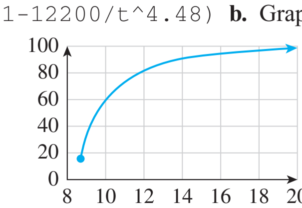


`x*(x<=0)+(x+1)*(0<x)*(x<=2)+x*(2<x)`

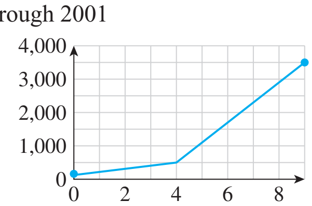
19. $f(6) \approx 2000$, $f(9) \approx 2800$, $f(7.5) \approx 2500$. In 1996, 2,000,000 SUVs were sold. In 1999, 2,800,000 were sold, and in the year beginning July, 1997, 2,500,000 were sold.
21. $f(6) - f(5)$; SUV sales increased more from 1995 to 1996 than from 1999 to 2000. **23. a.** [−1.5, 1.5] **b.** $N(-0.5) \approx 131$, $N(0) \approx 132$, $N(1) \approx 132$. In July 1999, approximately 131 million people were employed. In January 2000 and January 2001, approximately 132 million people were employed. **c.** [0.5, 1.5]; Employment was falling during the period July 2000–July 2001. **25. a.** (C) **b.** \$20.80 per shirt if the team buys 70 shirts. Graph:



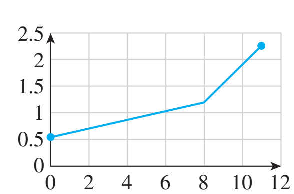
27. A quadratic model (B) is the best choice; the other models either predict perpetually increasing value of the euro or perpetually decreasing value of the euro.
29. a. `100*(1-12200/t^4.48)` **b.** Graph:



c. 82% **d.** 14 months
31. Midway through 2001



33. a. `(0.08*t+0.6)*(t<8)+(0.355*t-1.6)*(t>=8)`
Graph:



b. 2001

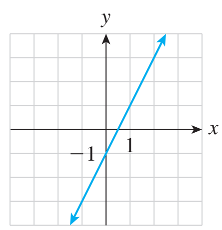
ANSWERS TO SELECTED EXERCISES

35. True. We can construct a table of values from any graph by reading off a set of values. **37.** False. In a numerically specified function, only certain values of the function are specified, giving only certain points on the graph. **39.** They are different portions of the graph of the associated equation $y = f(x)$. **41.** The graph of $g(x)$ is the same as the graph of $f(x)$, but shifted 5 units to the right.

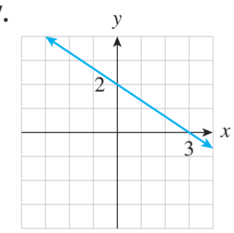
Section 1.3

1. Missing value: 11; $m = 3$ **3.** Missing value: -4; $m = -1$ **5.** Missing value: 7; $m = 3/2$ **7.** $f(x) = -x/2 - 2$ **9.** $f(0) = -5$, $f(x) = -x - 5$ **11.** f is linear: $f(x) = 4x + 6$ **13.** g is linear: $g(x) = 2x - 1$ **15.** $-3/2$ **17.** $1/6$ **19.** Undefined **21.** 0 **23.** $-4/3$

25.



27.



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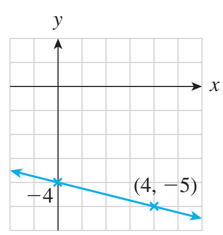

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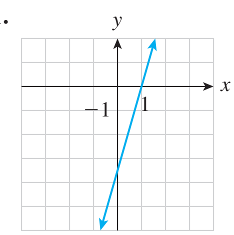

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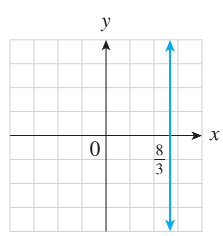

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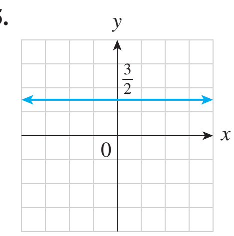
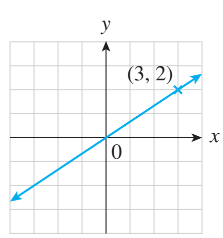


37.



39. 2 **41.** 2 **43.** -2 **45.** Undefined **47.** 1.5 **49.** -0.09 **51.** 1/2 **53.** $(d - b)/(c - a)$ **55. a.** 1 **b.** 1/2 **c.** 0 **d.** 3 **e.** $-1/3$ **f.** -1 **g.** Undefined **h.** $-1/4$ **i.** -2 **57.** $y = 3x$ **59.** $y = \frac{1}{4}x - 1$ **61.** $y = 10x - 203.5$ **63.** $y = -5x + 6$ **65.** $y = -3x + 2.25$ **67.** $y = -x + 12$ **69.** $y = 2x + 4$ **71.** Compute the corresponding successive changes Δx in x and Δy in y, and compute the ratios $\Delta y/\Delta x$. If the answer is always the same number, then the values in the table come from a linear function. **73.** $f(x) = -\frac{a}{b}x + \frac{c}{b}$. If $b = 0$, then $\frac{a}{b}$ is undefined, and y cannot be specified as a function of x. (The graph of the resulting equation would be a vertical line.) **75.** slope, 3 **77.** If m is positive then y will increase as x increases; if m is negative then y will decrease as x increases; if m is zero then y will not change as x changes. **79.** The slope increases, since an increase in the y-coordinate of the second point increases Δy while leaving Δx fixed.

Section 1.4

1. $C(x) = 1500x + 1200$ per day **a.** \$5700 **b.** \$1500 **c.** \$1500 **3.** Fixed cost = \$8000, marginal cost = \$25 per bicycle **5. a.** $C(x) = 0.4x + 70$, $R(x) = 0.5x$, $P(x) = 0.1x - 70$ **b.** $P(500) = -20$; a loss of \$20 **c.** 700 copies **7.** $q = -40p + 2000$ **9. a.** $q = -p + 156.4$; 53.4 million phones **b.** \$1, 1 million **11. a.** Demand: $q = -60p + 150$; supply: $q = 80p - 60$ **b.** \$1.50 each **13. a.** (1996, 125) and (1997, 135) or (1998, 140) and (1999, 150). **b.** The number of new in-ground pools increased most rapidly during the periods 1996–1997 and 1998–1999, when it rose by 10,000 new pools in a year. **15.** $N = 400 + 50t$ million transactions. The slope gives the additional number of online shopping transactions per year, and is measured in (millions of) transactions per year. **17. a.** $s = 14.4t + 240$**;** Medicare spending is predicted to rise at a rate of \$14.4 billion per year **b.** \$816 billion **19. a.** 2.5 ft/sec **b.** 20 feet along the track **c.** after 6 seconds **21. a.** 130 miles per hour **b.** $s = 130t - 1300$ **c.** After 5 seconds **23.** $F = 1.8C + 32$; 86°F; 72°F; 14°F; 7°F **25.** $I(N) = 0.05N + 50{,}000$; $N = \$1{,}000{,}000$; marginal income is $m = 5¢$ per dollar of net profit. **27.** $w = 2n - 58$; 42 billion pounds **29.** $c = 0.075m - 1.5$; 0.75 pounds **31.** $T(r) = (1/4)r + 45$; $T(100) = 70°\text{F}$ **33.** $P(x) = 100x - 5132$, with domain [0, 405]. For profit, $x \geq 52$ **35.** 5000 units **37.** $FC/(SP - VC)$ **39.** $P(x) = 579.7x - 20{,}000$, with domain $x \geq 0$; $x = 34.50$ g per day for break even **41.** Increasing by \$355,000 per year **43. a.** $y = -30t + 200$ **b.** $y = 50t - 200$

c. $y = \begin{cases} -30t + 200 & \text{if } 0 \leq t \leq 5 \\ 50t - 200 & \text{if } 5 < t \leq 9 \end{cases}$ **d.** 150

45. $C(t) = \begin{cases} -1{,}400t + 30{,}000 & \text{if } 0 \leq t \leq 5 \\ 7{,}400t - 14{,}000 & \text{if } 5 < t \leq 10 \end{cases}$

$C(3) = 25{,}800$ students

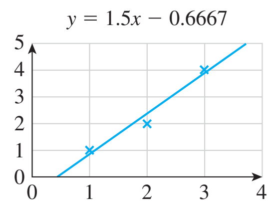
47. $d(r) = \begin{cases} -40r + 74 & \text{if } 1.1 \leq r \leq 1.3 \\ \dfrac{130r}{3} - \dfrac{103}{3} & \text{if } 1.3 < r \leq 1.6 \end{cases}$; $d(1) = 34\%$

49. Bootlags per zonar; bootlags **51.** It must increase by 10 units each day, including the third. **53.** (B) **55.** Increasing the number of items from the breakeven results in a profit: Because the slope of the revenue graph is larger than the slope of the cost graph, it is higher than the cost graph to the right of the point of intersection, and hence corresponds to a profit.

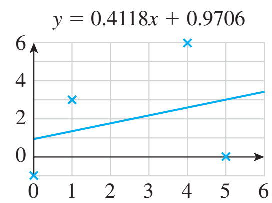
Section 1.5

1. 6 **3.** 86 **5. a.** 0.5 (better fit) **b.** 0.75 **7. a.** 27.42 **b.** 27.16 (better fit)

9.



11.



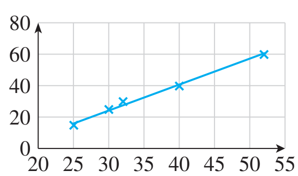
13. a. $r = 0.9959$ (best, not perfect) **b.** $r = 0.9538$ **c.** $r = 0.3273$ (worst)

15.

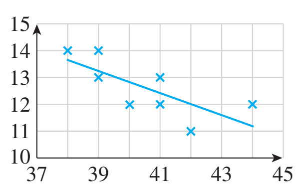
	x	y	xy	x^2
	3	500	1500	9
	5	600	3000	25
	7	800	5600	49
Totals	15	1900	10100	83

$y = 75x + 258.33$; 858.33 million

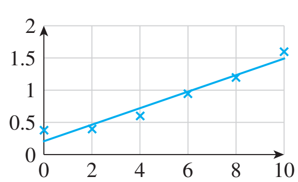
17. $y = 2.5t + 5.67$; \$13.17 billion **19.** $y = 0.135x + 0.15$; 6.9 million jobs **21. a.** $y = 1.62x - 23.87$. Graph:



b. Each acre of cultivated land produces about 1.62 tons of soybeans **23. a.** Regression line: $y = -0.40x + 29$. Graph:

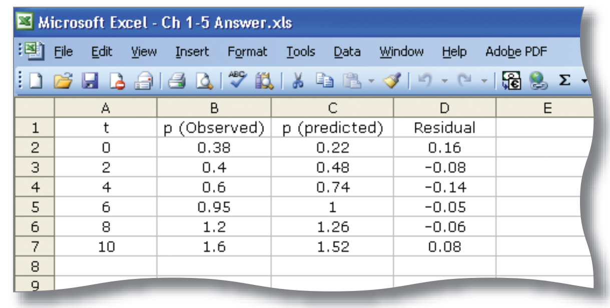


The graph suggests a relationship between x and y. **b.** The poverty rate declines by 0.40% for each \$1000 increase in the median household income. **c.** $r \approx -0.7338$; not a strong correlation **25. a.** $p = 0.13t + 0.22$. Graph:



b. Yes; the first and last points lie above the regression line, while the central points lie below it, suggesting a curve.

c.



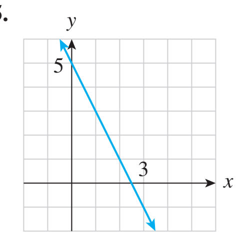
	A	B	C	D	E
1	t	p (Observed)	p (predicted)	Residual	
2	0	0.38	0.22	0.16	
3	2	0.4	0.48	-0.08	
4	4	0.6	0.74	-0.14	
5	6	0.95	1	-0.05	
6	8	1.2	1.26	-0.06	
7	10	1.6	1.52	0.08	
8					
9					

Notice that the residuals are positive at first, then become negative, and then become positive, confirming the impression from the graph. **27.** The line that passes through (a, b) and (c, d) gives a sum-of-squares error SSE = 0, which is the smallest value possible. **29.** The regression line is the line passing through the given points. **31.** 0 **33.** No. The regression line through (−1, 1), (0, 0), and (1, 1) passes through none of these points.

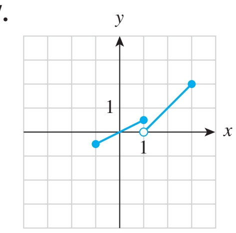
Chapter 1 Review

1. a. 1 **b.** −2 **c.** 0 **d.** −1 **3. a.** 1 **b.** 0 **c.** 0 **d.** −1

5.



7.



9. Absolute value **11.** Linear **13.** Quadratic **15.** $y = -x + 1$ **17.** $y = (1/2)x - 1$ **19.** The first line, $y = x + 1$, is the better fit. **21.** $y \approx 0.857x + 1.24$, $r \approx 0.92$ **23. a.** (A) **b.** (A) Leveling off (B) Rising (C) Rising; they begin to fall after 7 months (D) Rising **25. a.** 2080 hits per day **b.** Probably not. This model predicts that Web site traffic will start to decrease as advertising increases beyond \$8500 per month, and then drop toward zero. **27. a.** $q = -60p + 950$ **b.** 50 novels per month **c.** \$10, for a profit of \$1200.

Chapter 2

Section 2.1

1. (2, 2) **3.** (3, 1) **5.** (6, 6) **7.** (5/3, −4/3) **9.** (0, −2) **11.** $(x, (1 - 2x)/3)$ or $\left(\frac{1}{2}(1 - 3y), y\right)$ **13.** No solution **15.** (5, 0) **17.** (0.3, −1.1) **19.** (116.6, −69.7) **21.** (3.3, 1.8) **23.** (3.4, 1.9) **25.** 200 quarts of vanilla and 100 quarts of mocha **27.** 2 servings of Mixed Cereal and 1 serving of Mango Tropical Fruit **29. a.** 4 servings of beans and 5 slices of bread **b.** No. One of the variables in the solution of the system has a negative value. **31.** Mix 5 servings of Cell-Tech and 6 servings of RiboForce HP for a cost of \$20.60. **33.** 100 CSCO, 150 NOK **35.** 100 ED, 200 KSE **37.** 242 in favor and 193 against

39. 5 soccer games and 7 football games **41.** 7 **43.** \$1.50 each **45.** 55 widgets **47.** Demand: $q = -4p + 47$; supply: $q = 4p - 29$; equilibrium price: \$9.50 **49.** 33 pairs of dirty socks and 11 T-shirts **51.** \$1200 **53.** A system of three equations in two unknowns will have a unique solution if either (1) the three corresponding lines intersect in a single point, or (2) two of the equations correspond to the same line, and the third line intersects it in a single point. **55.** Yes. Even if two lines have negative slope, they will still intersect if the slopes differ. **57.** You cannot round both of them up, because there will not be sufficient eggs and cream. Rounding both answers down will ensure that you will not run out of ingredients. It may be possible to round one answer down and the other up, and this should be tried. **59.** (B) **61.** (B) **63.** Answers will vary. **65.** It is very likely. Two randomly chosen straight lines are unlikely to be parallel.

Section 2.2

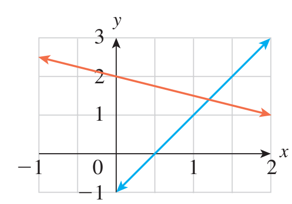
1. (3, 1) **3.** (6, 6) **5.** $\left(\frac{1}{2}(1-3y), y\right)$, y arbitrary **7.** No solution **9.** $(1/4, 3/4)$ **11.** No solution **13.** $(10/3, 1/3)$ **15.** $(4, 4, 4)$ **17.** $\left(-1, -3, \frac{1}{2}\right)$ **19.** (z, z, z), z arbitrary **21.** No solution **23.** $(-1, 1, 1)$ **25.** $(1, z-2, z)$, z arbitrary **27.** $(4+y, y, -1)$, y arbitrary **29.** $(4 - y/3 + z/3, y, z)$, y arbitrary, z arbitrary **31.** $(-17, 20, -2)$ **33.** $\left(-\frac{3}{2}, 0, \frac{1}{2}, 0\right)$ **35.** $(-3z, 1-2z, z, 0)$, z arbitrary **37.** $\left(\frac{1}{5}(7 - 17z + 8w), \frac{1}{5}(1 - 6z - 6w), z, w\right)$, z, w arbitrary **39.** $(1, 2, 3, 4, 5)$ **41.** $(-2, -2 + z - u, z, u, 0)$ z, u arbitrary **43.** $(16, 12/7, -162/7, -88/7)$ **45.** $(-8/15, 7/15, 7/15, 7/15, 7/15)$ **47.** $(1.0, 1.4, 0.2)$ **49.** $(-5.5, -0.9, -7.4, -6.6)$ **51.** A pivot is an entry in a matrix that is selected to "clear a column;" that is, use the row operations of a certain type to obtain zeros everywhere above and below it. "Pivoting" is the procedure of clearing a column using a designated pivot. **53.** $2R_1 + 5R_4$, or $6R_1 + 15R_4$ (which is less desirable). **55.** It will include a row of zeros. **57.** The claim is wrong. If there are more equations than unknowns, there can be a unique solution as well as row(s) of zeros in the reduced matrix, as in Example 6. **59.** Two **61.** The number of pivots must equal the number of variables, because no variable will be used as a parameter. **63.** A simple example is: $x = 1$; $y - z = 1$; $x + y - z = 2$.

Section 2.3

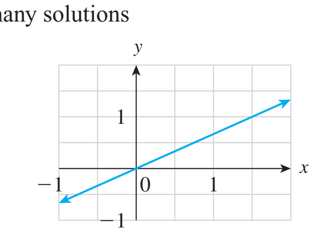
1. 100 batches of vanilla, 50 batches of mocha, and 100 batches of strawberry **3.** 3 sections of Finite Math, 2 sections of Applied Calculus and 1 section of Computer Methods **5.** 5 of each **7.** 22 tons from Cheesy Cream, 56 tons from Super Smooth & Sons, and 22 tons from Bagel's Best Friend **9.** 10 evil sorcerers, 50 trolls, and 500 orcs **11.** \$3.6 billion for rock music, \$1.8 billion for rap music, and \$0.4 billion for classical music. **13.** It donated \$600 to each of the MPBF and the SCN, and \$1200 to the Jets. **15.** United: 120; American: 40; SouthWest: 50 **17.** \$5000 in PNF, \$2000 in FDMMX, \$2000 in FFLIX **19.** 100 APPL, 20 HPQ, 80 DELL **21.** Microsoft: 88 million, Time Warner: 79 million, Yahoo: 75 million, Google: 42 million **23.** The third equation is $x + y + z + w = 1$. General Solution: $x = -1.58 + 3.89w$, $y = 1.63 - 2.99w$, $z = 0.95 - 1.9w$, w arbitrary. State Farm is most impacted by Other. **25. a.** Brooklyn to Manhattan: 500 books; Brooklyn to Long Island: 500 books; Queens to Manhattan: 1000 books; Queens to Long Island: 1000 books. **b.** Brooklyn to Manhattan: 1000 books; Brooklyn to Long Island: none; Queens to Manhattan: 500 books; Queens to Long Island: 1500 books, giving a total cost of \$8000. **27. a.** The associated system of equations has infinitely many solutions. **b.** No; the associated system of equations still has infinitely many solutions. **c.** Yes; North America to Australia: 440,000, North America to South Africa: 190,000, Europe to Australia: 950,000, Europe to South Africa: 950,000. **29. a.** $x + y = 14{,}000$; $z + w = 95{,}000$; $x + z = 63{,}550$; $y + w = 45{,}450$. The system does not have a unique solution, indicating that the given data are insufficient to obtain the missing data. **b.** $(x, y, z, w) = (5600, 8400, 57{,}950, 37{,}050)$ **31. a.** No; The general solution is: Eastward Blvd.: $S + 200$; Northwest La.: $S + 50$; Southwest La.: S, where S is arbitrary. Thus it would suffice to know the traffic along Southwest La. **b.** Yes, as it leads to the solution Eastward Blvd.: 260; Northwest La.: 110; Southwest La.: 60 **c.** 50 vehicles per day **33. a.** No; the corresponding system of equations is underdetermined. The net flow of traffic along any of the three stretches of Broadway would suffice. **b.** West **35.** \$10 billion **37.** x = Water, y = Gray matter, z = Tumor **39.** x = Water, y = Bone, z = Tumor, u = Air **41.** Tumor **43.** 200 Democrats, 20 Republicans, 13 of other parties **45.** Yes; \$20m in Company X; \$5m in Company Y, \$10m in Company Z, and \$30m in Company W **47.** It is not realistic to expect to use exactly all of the ingredients. Solutions of the associated system may involve negative numbers or not exist. Only solutions with nonnegative values for all the unknowns correspond to being able to use up all of the ingredients. **49.** Yes; $x = 100$ **51.** Yes; $0.3x - 0.7y + 0.3z = 0$ is one form of the equation. **53.** No; represented by an inequality rather than an equation. **55.** Answers will vary.

Chapter 2 Review

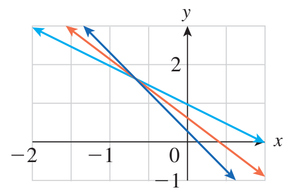
1. One solution



3. Infinitely many solutions



5. One solution.



7. $(6/5, 7/5)$ **9.** $(3y/2, y)$, y arbitrary **11.** $(-0.7, 1.7)$ **13.** $(-1, -1, -1)$ **15.** $(z-2, 4(z-1), z)$, z arbitrary **17.** No solution **19. a.** $-40°$ **b.** 320°F (160°C) **c.** It is impossible; setting $F = 1.8C$ leads to an inconsistent system of equations. **21.** 550 packages from Duffin House, 350 from Higgins Press **23.** 600 packages from Duffin House, 200 from Higgins Press **25.** \$40 **27.** 7 of each **29.** 5000 hits per day at OHaganBooks.com, 1250 at JungleBooks.com, 3750 at FarmerBooks.com **31.** DHS: 1000 shares, HPR: 600 shares, SPUB: 400 shares. **33. a.** $x = 100, y = 100 + w, z = 300 - w$, w arbitrary **b.** 100 book orders per day **c.** 300 book orders per day **d.** $x = 100$, $y = 400$, $z = 0$, $w = 300$ **e.** 100 book orders per day **35.** New York to OHaganBooks.com: 450 packages, New York to FantasyBooks.com: 50 packages, Illinois to OHaganBooks.com: 150 packages, Illinois to FantasyBooks.com: 150 packages.

Chapter 3

Section 3.1

1. 1×4; 0 **3.** 4×1; 5/2 **5.** $p \times q$; e_{22} **7.** 2×2; 3 **9.** $1 \times n$; d_r **11.** $x = 1, y = 2, z = 3, w = 4$

13. $\begin{bmatrix} 0.25 & -2 \\ 1 & 0.5 \\ -2 & 5 \end{bmatrix}$ **15.** $\begin{bmatrix} -0.75 & -1 \\ 0 & -0.5 \\ -1 & 6 \end{bmatrix}$

17. $\begin{bmatrix} -1 & -1 \\ 1 & -1 \\ -1 & 5 \end{bmatrix}$ **19.** $\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 4 \end{bmatrix}$

21. $\begin{bmatrix} 4 & -1 & -1 \\ 5 & 1 & 0 \end{bmatrix}$ **23.** $\begin{bmatrix} -2+x & 0 & 1+w \\ -5+z & 3+r & 2 \end{bmatrix}$

25. $\begin{bmatrix} -1 & -2 & 1 \\ -5 & 5 & -3 \end{bmatrix}$ **27.** $\begin{bmatrix} 9 & 15 \\ 0 & -3 \\ -3 & 3 \end{bmatrix}$

29. $\begin{bmatrix} -8.5 & -22.35 & -24.4 \\ 54.2 & 20 & 42.2 \end{bmatrix}$

31. $\begin{bmatrix} 1.54 & 8.58 \\ 5.94 & 0 \\ 6.16 & 7.26 \end{bmatrix}$ **33.** $\begin{bmatrix} 7.38 & 76.96 \\ 20.33 & 0 \\ 29.12 & 39.92 \end{bmatrix}$

35. $\begin{bmatrix} -19.85 & 115.82 \\ -50.935 & 46 \\ -57.24 & 94.62 \end{bmatrix}$ **37. a.** [720 680 350]

b. [760 800 300] **39.** Sales $= \begin{bmatrix} 700 & 1300 & 2000 \\ 400 & 300 & 500 \end{bmatrix}$

Inventory − Sales $= \begin{bmatrix} 300 & 700 & 3000 \\ 600 & 4700 & 1500 \end{bmatrix}$

41. Profit = Revenue − Cost;

	2004	*2005*	*2006*
Full Boots	\$8000	\$7200	\$8800
Half Boots	\$5600	\$5760	\$7040
Sandals	\$2800	\$3500	\$4000

43. 1980 distribution $= A =$ [49.1 58.9 75.4 43.2]; 1990 distribution $= B =$ [50.8 59.7 85.4 52.8]; Net change 1980 to 1990 $= B - A =$ [1.7 0.8 10 9.6] (all net increases) **45.** Total Bankruptcy Filings = Filings in Manhattan + Filings in Brooklyn + Filings in Newark = [150 250 150 100 150] + [300 400 300 200 250] + [250 400 250 200 200] = [700 1050 700 500 600] **47.** Filings in Brooklyn − Filings in Newark = [300 400 300 200 250] − [250 400 250 200 200] = [50 0 50 0 50]. The difference was greatest in January 01, July 01, and January 02.

49. a. Use =

	Proc	Mem	Tubes
Pom II	2	16	20
Pom Classic	1	4	40

Inventory $= \begin{bmatrix} 500 & 5000 & 10{,}000 \\ 200 & 2000 & 20{,}000 \end{bmatrix}$

Inventory $- 100 \cdot$ Use $= \begin{bmatrix} 300 & 3400 & 8000 \\ 100 & 1600 & 16{,}000 \end{bmatrix}$

b. After 4 months.

51. a. $A = \begin{bmatrix} 440 & 190 \\ 950 & 950 \\ 1790 & 200 \end{bmatrix}$ $D = \begin{bmatrix} -20 & 40 \\ 50 & 50 \\ 0 & 100 \end{bmatrix}$

2008 Tourism $= A + D = \begin{bmatrix} 420 & 230 \\ 1000 & 1000 \\ 1790 & 300 \end{bmatrix}$

b. $\frac{1}{2}(A + B)$; $\begin{bmatrix} 430 & 210 \\ 975 & 975 \\ 1790 & 250 \end{bmatrix}$

53. The ijth entry of the sum $A + B$ is obtained by adding the ijth entries of A and B. **55.** It would have zeros down the main diagonal: $A = \begin{bmatrix} 0 & \# & \# & \# & \# \\ \# & 0 & \# & \# & \# \\ \# & \# & 0 & \# & \# \\ \# & \# & \# & 0 & \# \\ \# & \# & \# & \# & 0 \end{bmatrix}$ The symbols # indicate arbitrary numbers. **57.** $(A^T)_{ij} = A_{ji}$ **59.** Answers will vary.

a. $\begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & -4 & 5 \\ 4 & 0 & 1 \\ -5 & -1 & 0 \end{bmatrix}$ **61.** The associativity of matrix addition is a consequence of the associativity of addition of numbers, since we add matrices by adding the corresponding entries (which are real numbers). **63.** Answers will vary.

Section 3.2

1. $[13]$ **3.** $[5/6]$ **5.** $[-2y+z]$ **7.** Undefined
9. $[3 \quad 0 \quad -6 \quad -2 \quad]$ **11.** $[-6 \quad 37 \quad 7]$

13. $\begin{bmatrix} -4 & -7 & -1 \\ 9 & 17 & 0 \end{bmatrix}$ **15.** $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **17.** $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ **19.** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

21. Undefined **23.** $\begin{bmatrix} 1 & -5 & 3 \\ 0 & 0 & 9 \\ 0 & 4 & 1 \end{bmatrix}$ **25.** $\begin{bmatrix} 3 \\ -4 \\ 0 \\ 3 \end{bmatrix}$

27. $\begin{bmatrix} 0.23 & 5.36 & -21.65 \\ -13.18 & -5.82 & -16.62 \\ -11.21 & -9.9 & 0.99 \\ -2.1 & 2.34 & 2.46 \end{bmatrix}$

29. $A^2 = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $A^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $A^{100} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

31. $\begin{bmatrix} 4 & -1 \\ -1 & -7 \end{bmatrix}$ **33.** $\begin{bmatrix} 4 & -1 \\ -12 & 2 \end{bmatrix}$ **35.** $\begin{bmatrix} -2 & 1 & -2 \\ 10 & -2 & 2 \\ -10 & 2 & -2 \end{bmatrix}$

37. $\begin{bmatrix} -2+x-z & 2-r & -6+w \\ 10+2z & -2+2r & 10 \\ -10-2z & 2-2r & -10 \end{bmatrix}$

39. a.–d. $P^2 = P^4 = P^8 = P^{1000} = \begin{bmatrix} 0.2 & 0.8 \\ 0.2 & 0.8 \end{bmatrix}$

41. a. $P^2 = \begin{bmatrix} 0.01 & 0.99 \\ 0 & 1 \end{bmatrix}$ **b.** $P^4 = \begin{bmatrix} 0.0001 & 0.9999 \\ 0 & 1 \end{bmatrix}$

c. and d. $P^8 \approx P^{1000} \approx \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

43. a.–d. $P^2 = P^4 = P^8 = P^{1000} = \begin{bmatrix} 0.25 & 0.25 & 0.50 \\ 0.25 & 0.25 & 0.50 \\ 0.25 & 0.25 & 0.50 \end{bmatrix}$

45. $2x - y + 4z = 3$; $-4x + 3y/4 + z/3 = -1$; $-3x = 0$
47. $x - y + w = -1$; $x + y + 2z + 4w = 2$

49. $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

51. $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ \frac{3}{4} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 1 \end{bmatrix}$

53. Revenue = Price × Quantity =

$$[15 \quad 10 \quad 12] \begin{bmatrix} 50 \\ 40 \\ 30 \end{bmatrix} = [1510]$$

55. Price: $\begin{matrix} \text{Hard} \\ \text{Soft} \\ \text{Plastic} \end{matrix} \begin{bmatrix} 30 \\ 10 \\ 15 \end{bmatrix}$;

$$\begin{bmatrix} 700 & 1300 & 2000 \\ 400 & 300 & 500 \end{bmatrix} \begin{bmatrix} 30 \\ 10 \\ 15 \end{bmatrix} = \begin{bmatrix} \$64{,}000 \\ \$22{,}500 \end{bmatrix}$$

57. Number of books = Number of books per editor × Number of editors $= [3 \quad 3.5 \quad 5 \quad 5.2] \begin{bmatrix} 16{,}000 \\ 15{,}000 \\ 12{,}500 \\ 13{,}000 \end{bmatrix} = 230{,}600$ new books

59. \$4300 billion (or \$4.3 trillion) **61.** $D = N(F - M)$ where N is the income per person, and F and M are, respectively, the female and male populations in 2005; \$140 billion. **63.** $[1.2 \quad 1.0]$, which represents the amount, in billions of pounds, by which cheese production in north central states exceeded that in western states. **65.** Number of bankruptcy filings handled by firm = Percentage handled by firm × Total number =

$$[0.10 \quad 0.05 \quad 0.20] \begin{bmatrix} 150 & 150 & 150 \\ 300 & 300 & 250 \\ 250 & 250 & 200 \end{bmatrix} = [80 \quad 80 \quad 67.5]$$

67. The number of filings in Manhattan and Brooklyn combined in each of the months shown.

69. $[1 \quad -1 \quad 1] \begin{bmatrix} 150 & 150 & 150 \\ 300 & 300 & 250 \\ 250 & 250 & 200 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [300]$

71. $\begin{bmatrix} 2 & 16 & 20 \\ 1 & 4 & 40 \end{bmatrix} \begin{bmatrix} 100 & 150 \\ 50 & 40 \\ 10 & 15 \end{bmatrix} = \begin{bmatrix} \$1200 & \$1240 \\ \$700 & \$910 \end{bmatrix}$

73. $AB = \begin{bmatrix} 29.6 \\ 85.5 \\ 97.5 \end{bmatrix}$ $AC = \begin{bmatrix} 22 & 7.6 \\ 47.5 & 38 \\ 89.5 & 8 \end{bmatrix}$ The entries of AB give the number of people from each of the three regions who settle in Australia or South Africa, while the entries in AC break those figures down further into settlers in South Africa and settlers in Australia. **75.** Distribution in 2003 $= A = [53.3 \quad 64.0 \quad 101.6 \quad 65.4]$; Distribution in 2004 $= A \cdot P \approx [53.1 \quad 63.9 \quad 102.0 \quad 65.3]$ **77.** Answers will vary. One example:

$A = [1 \quad 2]$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Another example: $A = [1]$, $B = [1 \quad 2]$. **79.** Multiplication of 1×1 matrices is just ordinary multiplication of the single entries: $[a][b] = [ab]$.
81. The claim is correct. Every matrix equation represents the equality of two matrices. Equating the corresponding entries gives a system of equations. **83.** Answers will vary. Here is a possible scenario: costs of items A, B and C in 1995 $= [10 \quad 20 \quad 30]$, percentage increases in these costs in 1996 $= [0.5 \quad 0.1 \quad 0.20]$, actual increases in costs $= [10 \times 0.5 \quad 20 \times 0.1 \quad 30 \times 0.20]$ **85.** It produces a matrix whose ij entry is the product of the ij entries of the two matrices.

Section 3.3

1. Yes **3.** Yes **5.** No **7.** $\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$ **9.** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ **13.** Singular **15.** $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

17. $\begin{bmatrix} 1 & -1 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}$ **19.** $\begin{bmatrix} 1 & \frac{1}{3} & -\frac{1}{3} \\ 1 & -\frac{2}{3} & -\frac{1}{3} \\ -1 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

21. Singular **23.** $\begin{bmatrix} 0 & 1 & -2 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 2 & -1 \\ 0 & 1 & -1 & 1 \end{bmatrix}$

25. $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **27.** -2; $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

29. -2; $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$ **31.** $1/36$; $\begin{bmatrix} 6 & 6 \\ 0 & 6 \end{bmatrix}$ **33.** 0; Singular

35. $\begin{bmatrix} 0.38 & 0.45 \\ 0.49 & -0.41 \end{bmatrix}$ **37.** $\begin{bmatrix} 0.00 & -0.99 \\ 0.81 & 2.87 \end{bmatrix}$ **39.** Singular

41. $\begin{bmatrix} 91.35 & -8.65 & 0 & -71.30 \\ -0.07 & -0.07 & 0 & 2.49 \\ 2.60 & 2.60 & -4.35 & 1.37 \\ 2.69 & 2.69 & 0 & -2.10 \end{bmatrix}$ **43.** $(5/2, 3/2)$

45. $(6, -4)$ **47.** $(6, 6, 6)$ **49. a.** $(10, -5, -3)$ **b.** $(6, 1, 5)$ **c.** $(0, 0, 0)$ **51. a.** 10/3 servings of beans, and 5/6 slices of bread **b.** $\begin{bmatrix} -1/2 & 1/6 \\ 7/8 & -5/24 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -A/2 + B/6 \\ 7A/8 - 5B/24 \end{bmatrix}$; that is, $-A/2 + B/6$ servings of beans and $7A/8 - 5B/24$ slices of bread **53. a.** 100 batches of vanilla, 50 batches of mocha, 100 batches of strawberry **b.** 100 batches of vanilla, no mocha, 200 batches of strawberry **c.** $\begin{bmatrix} 1 & -1/3 & -1/3 \\ -1 & 0 & 1 \\ 0 & 2/3 & -1/3 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix}$, or $A - B/3 - C/3$ batches of Vanilla, $-A + C$ batches of mocha, and $2B/3 - C/3$ batches of strawberry **55.** \$5000 in PNF, \$2000 in FDMMX, \$2000 in FFLIX **57.** 100 APPL, 20 HPQ, 80 DELL **59.** Distribution in 2003 $= A =$ [53.3 64.0 101.6 65.4]; Distribution in 2002 $= A \cdot P^{-1} \approx$ [53.5 64.1 101.2 65.5] **61. a.** $(-0.7071, 3.5355)$ **b.** R^2, R^3 **c.** R^{-1} **63.** [37 81 40 80 15 45 40 96 29 59 4 8] **65.** CORRECT ANSWER **67.** (A) **69.** The inverse does not exist—the matrix is singular. (If two rows of a matrix are the same, then row reducing it will lead to a row of zeros, and so it cannot be reduced to the identity.) **73.** When one or more of the d_i are zero. If that is the case, then the matrix $[D \mid I]$ easily reduces to a matrix that has a row of zeros on the left-hand portion, so that D is singular, Conversely, if none of the d_i are zero, then $[D \mid I]$ easily reduces to a matrix of the form $[I \mid E]$, showing that D is invertible.

75. $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$ **77.** If A has an inverse, then every system of equations $AX = B$ has a unique solution, namely $X = A^{-1}B$. But if A reduces to a matrix with a row of zeros, then such a system has either infinitely many solutions or no solution at all.

Section 3.4

1.

A \ **B**	p	r
a	1	10
b	2	−4

3.

A \ **B**	c
3	[−1]

5.

A \ **B**	b
q	[0]

7. Strictly determined. A's optimal strategy is a; B's optimal strategy is q; value: 1 **9.** Not strictly determined **11.** Not strictly determined **13.** -1 **15.** -0.25 **17.** $[0 \;\; 0 \;\; 1 \;\; 0]$; $e = 2.25$ **19.** $[1 \;\; 0 \;\; 0]^T$ or $[0 \;\; 1 \;\; 0]^T$; $e = 1/4$ **21.** $R = [1/4 \;\; 3/4]$, $C = [3/4 \;\; 1/4]^T$, $e = -1/4$ **23.** $R = [3/4 \;\; 1/4]$, $C = [3/4 \;\; 1/4]^T$, $e = -5/4$

25.

You \ Friend	H	T
H	−1	1
T	1	−1

27. F = France; S = Sweden; N = Norway;

You Invade \ **Your Opponent Defends**	F	S	N
F	−1	1	1
S	1	−1	1
N	1	1	−1

29. B = Brakpan; N = Nigel; S = Springs;

You \ **Your Opponent**	B	N	S
B	0	0	1000
N	0	0	1000
S	−1000	−1000	0

31. P = PleasantTap; T = Thunder Rumble; S = Strike the Gold, N = None;

You Bet \ **Winner**	P	T	S	N
P	25	−10	−10	−10
T	−10	35	−10	−10
S	−10	−10	40	−10

33. a. CE should charge \$1000 and GCS should charge \$900; 15% gain in market share for CE **b.** CE should charge \$1200 (the more CE can charge for the same market, the better!) **35.** Pablo vs. Noto; evenly matched **37.** Both commanders should use the northern route; 1 day **39.** Confess

41. a.

Bush \ **Kerry**	F	O
F	24	21
O	25	24

b. Both candidates should visit Ohio, leaving Bush with a 21% chance of winning the election. **43.** You can expect to lose 39 customers. **45.** Option 2: move to the suburbs. **47. a.** About 66% **b.** Yes; spend the whole night studying game theory; 75% **c.** Game theory; 57.5% **49. a.** Lay off 10 workers; Cost: \$40,000 **b.** 60 inches of snow, costing \$350,000 **c.** Lay off 15 workers **51.** Allocate 1/7 of the budget to WISH and the rest (6/7) to WASH. Softex will lose approximately \$2860. **53.** Like a saddle point in a payoff matrix, the center of a saddle is a low point (minimum height) in one direction and a high point (maximum) in a perpendicular direction. **55.** Although there is a saddle point in the 2,4 position, you would be wrong to use saddle points (based on the minimax criterion) to reach the conclusion that row strategy 2 is best. One reason is that the entries in the matrix do not represent payoffs, because high numbers of employees in an area do not necessarily represent benefit to the row player. Another reason for this is that there is no opponent deciding what your job will be in such a way as to force you into the least populated job. **57.** If you strictly alternate the two strategies the column player will know which pure strategy you will play on each move, and can choose a pure strategy accordingly. For example, consider the game $\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$. By the analysis of Example 3 (or the symmetry of the game), the best strategy for the row player is $[0.5 \quad 0.5]$ and the best strategy for the column player is $[0.5 \quad 0.5]^T$. This gives an expected value of 0.5 for the game. However, suppose that the row player alternates A and B strictly and that the column player catches on to this. Then, whenever the row player plays A the column player will play b and whenever the row player plays B the column player will play a. This gives a payoff of 0 each time, worse for the row player than the expected value of 0.5.

Section 3.5

1. a. 0.8 **b.** 0.2 **c.** 0.05 **3.** $\begin{bmatrix} 0.2 & 0.1 \\ 0.5 & 0 \end{bmatrix}$
5. $[52{,}000 \quad 40{,}000]^T$ **7.** $[50{,}000 \quad 50{,}000]^T$
9. $[2560 \quad 2800 \quad 4000]^T$
11. $[27{,}000 \quad 28{,}000 \quad 17{,}000]^T$ **13.** Increase of 100 units in each sector. **15.** Increase of $[1.5 \quad 0.2 \quad 0.1]^T$; the ith column of $(I - A)^{-1}$ gives the change in production necessary to meet an increase in external demand of one unit for the product of Sector i.
17. $A = \begin{bmatrix} 0.2 & 0.4 & 0.5 \\ 0 & 0.8 & 0 \\ 0 & 0.2 & 0.5 \end{bmatrix}$ **19.** Main DR: \$80,000, Bits & Bytes: \$38,000 **21.** Equipment Sector production approximately \$86,000 million, Components Sector production approximately \$140,000 million **23. a.** 0.006 **b.** textiles; clothing and footwear
25. Columns of $\begin{bmatrix} 1140.99 & 2.05 & 13.17 & 20.87 \\ 332.10 & 1047.34 & 26.05 & 111.18 \\ 0.12 & 0.13 & 1031.19 & 1.35 \\ 93.88 & 95.69 & 215.50 & 1016.15 \end{bmatrix}$ (in millions of dollars) **27. a.** \$0.78 **b.** Other food products **29.** It would mean that all of the sectors require neither their own product or the product of any other sector. **31.** It would mean that all of the output of that sector was used internally in the economy; none of the output was available for export and no importing was necessary. **33.** It means that an increase in demand for one sector (the column sector) has no effect on the production of another sector (the row sector). **35.** Usually, to produce one unit of one sector requires less than one unit of input from another. We would expect then that an increase in demand of one unit for one sector would require a smaller increase in production in another sector.

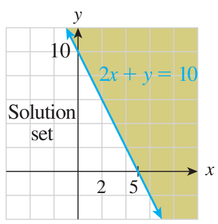
Chapter 3 Review

1. Undefined **3.** $\begin{bmatrix} 1 & 8 \\ 5 & 11 \\ 6 & 13 \end{bmatrix}$ **5.** $\begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 3 & 3 \end{bmatrix}$ **7.** $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$
9. $\begin{bmatrix} 2 & 4 \\ 1 & 12 \end{bmatrix}$ **11.** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **13.** $\begin{bmatrix} 1 & -1/2 & -5/2 \\ 0 & 1/4 & -1/4 \\ 0 & 0 & 1 \end{bmatrix}$
15. Singular **17.** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$; $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$
19. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$; $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$
21. $R = [1 \quad 0 \quad 0]$, $C = [0 \quad 1 \quad 0 \quad 0]^T$, $e = 1$
23. $R = [0 \quad 0.8 \quad 0.2]$, $C = [0.2 \quad 0 \quad 0.8]$, $e = -0.2$
25. $\begin{bmatrix} 1100 \\ 700 \end{bmatrix}$ **27.** $\begin{bmatrix} 48{,}125 \\ 22{,}500 \\ 10{,}000 \end{bmatrix}$
29. Inventory − Sales $= \begin{bmatrix} 2500 & 4000 & 3000 \\ 1500 & 3000 & 1000 \end{bmatrix} - \begin{bmatrix} 300 & 500 & 100 \\ 100 & 450 & 200 \end{bmatrix} = \begin{bmatrix} 2200 & 3500 & 2900 \\ 1400 & 2550 & 800 \end{bmatrix}$
31. Revenue = Quantity × Price $= \begin{bmatrix} 280 & 550 & 100 \\ 50 & 500 & 120 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \\ 5.5 \end{bmatrix} = \begin{bmatrix} 5250 \\ 3910 \end{bmatrix}$ Texas / Nevada
33. $[2000 \quad 4000 \quad 4000]\begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.4 & 0.6 & 0 \\ 0.2 & 0 & 0.8 \end{bmatrix} = [4000 \quad 2600 \quad 3400]$ **35.** Here are three. (1) It is possible for someone to be a customer at two different enterprises. (2) Some customers may stop using all three of the companies. (3) New customers can enter the field. **37.** Loss = Number of shares × (Purchase price − Dividends − Selling price) = $[1000 \quad 2000 \quad 2000]\left(\begin{bmatrix} 20 \\ 10 \\ 5 \end{bmatrix} - \begin{bmatrix} 0.10 \\ 0.10 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}\right)$ $= [42{,}700]$ **39.** Go with the “3 for 1” promotion and gain 20,000 customers from JungleBooks **41.** $A = \begin{bmatrix} 0.1 & 0.5 \\ 0.01 & 0.05 \end{bmatrix}$
43. \$1190 worth of paper, \$1802 worth of books.

Chapter 4

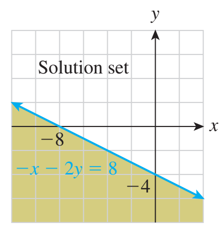
Section 4.1

1.



Unbounded

3.



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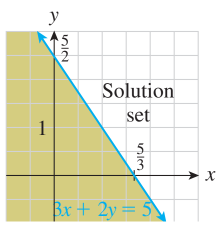
5.



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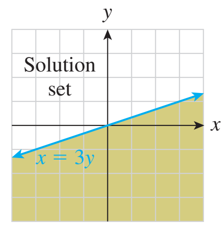
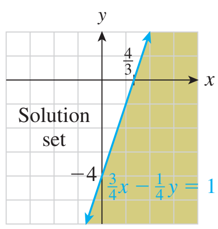


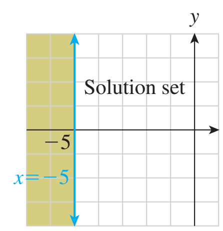
Unbounded

9.



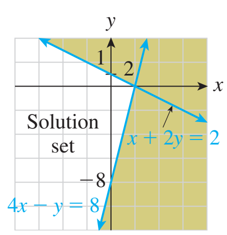
Unbounded

11.



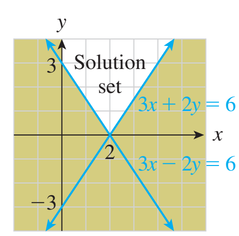
Unbounded

13.



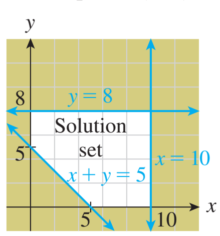
Unbounded;
Corner point: (2, 0)

15.



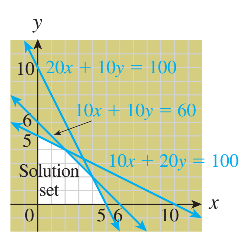
Unbounded;
Corner points: (2, 0), (0, 3)

17.



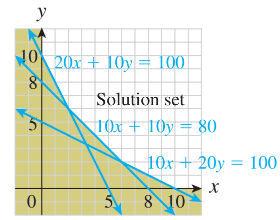
Bounded; Corner points:
(5, 0), (10, 0), (10, 8),
(0, 8), (0, 5)

19.



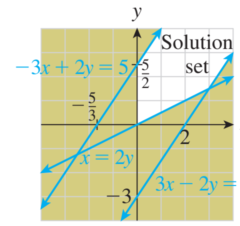
Bounded; Corner points:
(0, 0), (5, 0), (0, 5),
(2, 4), (4, 2)

21.



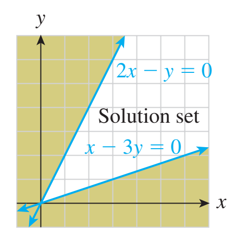
Unbounded; Corner points:
(0, 10), (10, 0), (2, 6), (6, 2)

23.



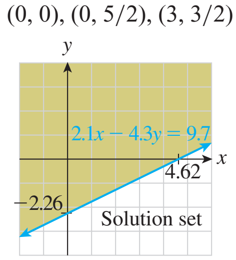
Unbounded;
Corner points:
(0, 0), (0, 5/2), (3, 3/2)

25.

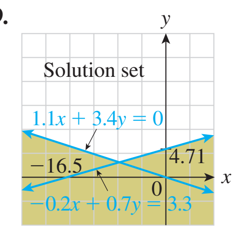


Unbounded; Corner point: (0, 0)

27.

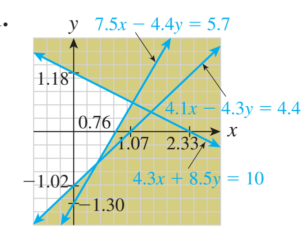


29.



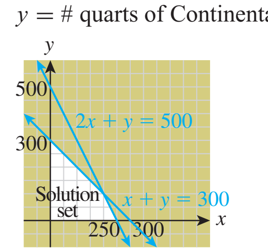
Corner point: (−7.74, 2.50)

31.



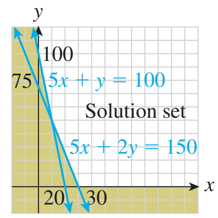
Corner points:
(0.36, −0.68), (1.12, 0.61)

33. x = # quarts of Creamy Vanilla,
y = # quarts of Continental Mocha



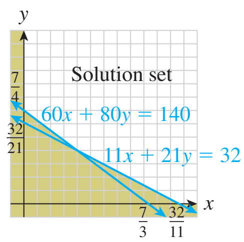
Corner points:
(0, 0), (250, 0), (0, 300), (200, 100)

35. x = # ounces of chicken, y = # ounces of grain



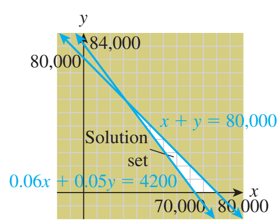
Corner points: (30, 0), (10, 50), (0, 100)

37. x = # servings of Mixed Cereal for Baby, y = # servings of Mango Tropical Fruit Dessert



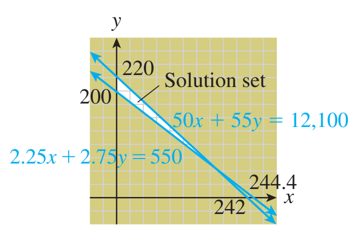
Corner points:
(0, 7/4), (1, 1), (32/11, 0)

39. x = # dollars in PNF, y = # dollars in FDMMX



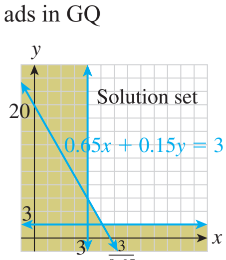
Corner points:
(70,000, 0), (80,000, 0), (20,000, 60,000)

41. x = # shares of MO, y = # shares of RAI



Corner points:
(0, 200), (0, 220), (220, 20)

43. x = # full-page ads in Sports Illustrated,
y = # full-page ads in GQ



Corner points: (3, 7), (4, 3) (Rounded)

45. An example is $x \geq 0, y \geq 0, x + y \geq 1$ **47.** The given triangle can be described as the solution set of the system $x \geq 0$, $y \geq 0$, $x + 2y \leq 2$. **49.** Answers may vary. One limitation is that the method is only suitable for situations with two unknown quantities. Accuracy is also limited when graphing. **51.** (C) **53.** (B) **55.** There are no feasible solutions; that is, it is impossible to satisfy all the constraints. **57.** Answers will vary.

Section 4.2

1. $p = 6, x = 3, y = 3$ **3.** $c = 4, x = 2, y = 2$ **5.** $p = 24$, $x = 7, y = 3$ **7.** $p = 16, x = 4, y = 2$ **9.** $c = 1.8, x = 6$, $y = 2$ **11.** Max: $p = 16, x = 4, y = 6$. Min: $p = 2, x = 2$, $y = 0$ **13.** No optimal solution; objective function unbounded **15.** $c = 28$; $(x, y) = (14, 0)$ and $(6, 4)$ and the line connecting them **17.** $c = 3, x = 3, y = 2$ **19.** No solution; feasible region empty **21.** You should make 200 quarts of vanilla and 100 quarts of mocha. **23.** Ruff, Inc., should use 100 oz of grain and no chicken. **25.** Feed your child 1 serving of cereal and 1 serving of dessert. **27.** Purchase 60 compact fluorescent light bulbs and 960 square feet of insulation for a saving of \$312 per year in energy costs. **29.** Mix 5 servings of Cell-Tech and 6 servings of RiboForce HP for a cost of \$20.60. **31.** Make 200 Dracula Salamis and 400 Frankenstein Sausages, for a profit of \$1400. **33.** Buy no shares of IBM and 500 shares of HPQ for maximum company earnings of \$600. **35.** Buy 220 shares of MO and 20 shares of RAI for a minimum total risk index is $c = 500$. **37.** Purchase 20 spots on "Becker" and 20 spots on "The Simpsons." **39.** He should instruct in diplomacy for 10 hours per week and in battle for 40 hours per week, giving a weekly profit of 2400 ducats. **41.** Gillian could expend a minimum of 360,000 pico-shirleys of energy by using 480 sleep spells and 160 shock spells. (There is actually a whole line of solutions joining the one above with $x = 2880/7, y = 1440/7$.) **43.** 100 hours per week for new customers and 60 hours per week for old customers. **45.** (A) **47.** Every point along the line connecting them is also an optimal solution. **49.** Answers will vary. **51.** Answers will vary. **53.** Answers will vary. A simple example is the following: Maximize profit $p = 2x + y$ subject to $x \geq 0, y \geq 0$. Then p can be made as large as we like by choosing large values of x and/or y. Thus there is no optimal solution to the problem. **55.** Mathematically, this means that there are infinitely many possible solutions: one for each point along the line joining the two corner points in question. In practice, select those points with integer solutions (because x and y must be whole numbers in this problem) that are in the feasible region and close to this line, and choose the one that gives the largest profit.

Section 4.3

1. $p = 8; x = 4, y = 0$ **3.** $p = 4; x = 4, y = 0$ **5.** $p = 80$; $x = 10, y = 0, z = 10$ **7.** $p = 53; x = 5, y = 0, z = 3$ **9.** $z = 14{,}500; x_1 = 0, x_2 = 500/3, x_3 = 5000/3$ **11.** $p = 6$; $x = 2, y = 1, z = 0, w = 3$ **13.** $p = 7; x = 1, y = 0, z = 2$, $w = 0, v = 4$ (or: $x = 1, y = 0, z = 2, w = 1, v = 3$.)

15. $p = 21$; $x = 0$, $y = 2.27$, $z = 5.73$ **17.** $p = 4.52$; $x = 1$, $y = 0$, $z = .67$, $w = 1.52$ **19.** $p = 7.7$; $x = 1.1$, $y = 0$, $z = 2.2$, $w = 0$, $v = 4$ **21.** You should purchase 500 calculus texts, no history texts and no marketing texts. The maximum profit is $5000 per semester. **23.** The company can make a maximum profit of $650 by making 100 gallons of PineOrange, 200 gallons of PineKiwi, and 150 gallons of OrangeKiwi. **25.** The department should offer no Ancient History, 30 sections of Medieval History, and 15 sections of Modern History, for a profit of $1,050,000. There will be 500 students without classes, but all sections and professors are used. **27.** Plant 80 acres of tomatoes and leave the other 20 acres unplanted. This will give you a profit of $160,000. **29.** It can make a profit of $10,000 by selling 1000 servings of Granola, 500 servings of Nutty Granola and no Nuttiest Granola. It is left with 2000 oz. almonds. **31.** Allocate 5 million gals to process A and 45 million gals to process C. Another solution: Allocate 10 million gals to process B and 40 million gals to process C. **33.** Use 15 servings of RiboForce HP and none of the others for a maximum of 75g creatine. **35.** She is wrong; you should buy 125 shares of IBM and no others. **37.** Allocate $2,250,000 to automobile loans, $500,000 to signature loans, and $2,250,000 to any combination of furniture loans and other secured loans. **39.** Invest $75,000 in Universal, none in the rest. Another optimal solution is: Invest $18,750 in Universal, and $75,000 in EMI. **41.** Tucson to Honolulu: 290 boards; Tucson to Venice Beach: 330 boards; Toronto to Honolulu: 0 boards; Toronto to Venice Beach: 200 boards, giving 820 boards shipped. **43.** Fly 10 people from Chicago to Los Angeles, 5 people from Chicago to New York, and 10 people from Denver to New York. **45.** Yes; the given problem can be stated as: Maximize $p = 3x - 2y$ subject to $-x + y - z \le 0$, $x - y - z \le 6$. **47.** The graphical method applies only to LP problems in two unknowns, whereas the simplex method can be used to solve LP problems with any number of unknowns. **49.** She is correct. Because there are only two constraints, there can only be two active variables, giving two or fewer nonzero values for the unknowns at each stage. **51.** A basic solution to a system of linear equations is a solution in which all the non-pivotal variables are taken to be zero; that is, all variables whose values are arbitrary are assigned the value zero. To obtain a basic solution for a given system of linear equations, one can row reduce the associated augmented matrix, write down the general solution, and then set all the parameters (variables with "arbitrary" values) equal to zero. **53.** No. Let us assume for the sake of simplicity that all the pivots are 1's. (They may certainly be changed to 1's without affecting the value of any of the variables.) Because the entry at the bottom of the pivot column is negative, the bottom row gets replaced by itself plus a positive multiple of the pivot row. The value of the objective function (bottom-right entry) is thus replaced by itself plus a positive multiple of the nonnegative rightmost entry of the pivot row. Therefore, it cannot decrease.

Section 4.4

1. $p = 20/3$; $x = 4/3$, $y = 16/3$ **3.** $p = 850/3$; $x = 50/3$, $y = 25/3$ **5.** $p = 750$; $x = 0$, $y = 150$, $z = 0$ **7.** $p = 135$; $x = 0, y = 25, z = 0, w = 15$ **9.** $c = 80; x = 20/3, y = 20/3$ **11.** $c = 100; x = 0, y = 100, z = 0$ **13.** $c = 111; x = 1, y = 1, z = 1$ **15.** $c = 200; x = 200, y = 0, z = 0, w = 0$ **17.** $p = 136.75; x = 0, y = 25.25, z = 0, w = 15.25$ **19.** $c = 66.67$; $x = 0$, $y = 66.67, z = 0$ **21.** $c = -250$; $x = 0$, $y = 500$, $z = 500$; $w = 1500$ **23.** Plant 100 acres of tomatoes and no other crops. This will give you a profit of $200,000. (You will be using all 100 acres of your farm.) **25.** 10 mailings to the East Coast, none to the Midwest, 10 to the West Coast. Cost: $900. Another solution resulting in the same cost is no mailings to the East Coast, 15 to the Midwest, none to the West Coast. **27.** 10,000 quarts of orange juice and 2000 quarts of orange concentrate **29.** Stock 10,000 rock CDs, 5000 rap CDs, and 5000 classical CDs for a maximum retail value of $255,000. **31.** One serving of cereal, one serving of juice, and no dessert! **33.** 15 bundles from Nadir, 5 from Sonny, and none from Blunt. Cost: $70,000. Another solution resulting in the same cost is 10 bundles from Nadir, none from Sonny, and 10 from Blunt. **35.** Mix 6 servings of Riboforce HP and 10 servings of Creatine Transport for a cost of $15.60. **37. a.** Build 1 convention-style hotel, 4 vacation-style hotels and 2 small motels. The total cost will amount to $188 million. **b.** Because 20% of this is $37.6 million, you will still be covered by the subsidy. **39.** Tucson to Honolulu: 500 boards/week; Tucson to Venice Beach: 120 boards/week; Toronto to Honolulu: 0 boards/week; Toronto to Venice Beach: 410 boards/week. Minimum weekly cost is $9700. **41.** $2500 from Congressional Integrity Bank, $0 from Citizens' Trust, $7500 from Checks R Us. **43.** Fly 10 people from Chicago to LA, 5 from Chicago to New York, none from Denver to LA, 10 from Denver to NY at a total cost of $4520. **45.** Hire no more cardiologists, 12 rehabilitation specialists, and 5 infectious disease specialists. **47.** The solution $x = 0$, $y = 0, \ldots,$ represented by the initial tableau may not be feasible. In phase I we use pivoting to arrive at a basic solution that is feasible. **49.** The basic solution corresponding to the initial tableau has all the unknowns equal to zero, and this is not a feasible solution because it does not satisfy the given inequality. **51.** (C) **53.** Answers may vary. Examples are Exercises 1 and 2. **55.** Answers may vary. A simple example is: Maximize $p = x + y$ subject to $x + y \le 10, x + y \ge 20, x \ge 0, y \ge 0$.

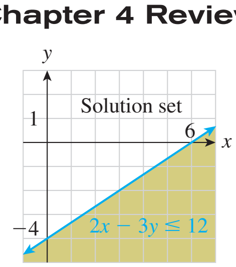
Section 4.5

1. Minimize $c = 6s + 2t$ subject to $s - t \ge 2, 2s + t \ge 1, s \ge 0, t \ge 0$ **3.** Maximize $p = 100x + 50y$ subject to $x + 2y \le 2, x + y \le 1, x \le 3, x \ge 0, y \ge 0$. **5.** Minimize $c = 3s + 4t + 5u + 6v$ subject to $s + u + v \ge 1, s + t + v \ge 1, s + t + u \ge 1, t + u + v \ge 1, s \ge 0, t \ge 0, u \ge 0, v \ge 0$. **7.** Maximize $p = 1000x + 2000y + 500z$ subject to $5x + z \le 1, -x + z \le 3, y \le 1, x - y \le 0, x \ge 0, y \ge 0, z \ge 0$. **9.** $c = 4; s = 2, t = 2$ **11.** $c = 80; s = 20/3, t = 20/3$ **13.** $c = 1.8; s = 6, t = 2$ **15.** $c = 25; s = 5, t = 15$ **17.** $c = 30; s = 30, t = 0, u = 0$ **19.** $c = 100; s = 0, t = 100, u = 0$ **21.** $c = 30; s = 10, t = 10, u = 10$ **23.** $R = [3/5 \quad 2/5], C = [2/5 \quad 3/5 \quad 0]^T, e = 1/5$ **25.** $R = [1/4 \quad 0 \quad 3/4], C = [1/2 \quad 0 \quad 1/2]^T, e = 1/2$ **27.** $R = [0 \quad 3/11 \quad 3/11 \quad 5/11]$, $C = [8/11 \quad 0 \quad 2/11 \quad 1/11]^T, e = 9/11$

29. 4 ounces each of fish and cornmeal, for a total cost of 40¢ per can; 5/12¢ per gram of protein, 5/12¢ per gram of fat. **31.** 100 oz of grain and no chicken, for a total cost of $1; 1/2¢ per gram of protein, 0¢ per gram of fat. **33.** One serving of cereal, one serving of juice, and no dessert! for a total cost of 37¢; 1/6¢ per calorie and 17/120¢ per % U.S. RDA of Vitamin C. **35.** 10 mailings to the East coast, none to the Midwest, 10 to the West Coast. Cost: $900; 20¢ per Democrat and 40¢ per Republican. OR 15 mailings to the Midwest and no mailing to the coasts. Cost: $900; 20¢ per Democrat and 40¢ per Republican. **37.** Gillian should use 480 sleep spells and 160 shock spells, costing 360,000 pico-shirleys of energy OR 2880/7 sleep spells and 1440/7 shock spells. **39.** T. N. Spend should spend about 73% of the days in Littleville, 27% in Metropolis, and skip Urbantown. T. L. Down should spend about 91% of the days in Littleville, 9% in Metropolis, and skip Urbantown. The expected outcome is that T. L. Down will lose about 227 votes per day of campaigning. **41.** Each player should show one finger with probability 1/2, two fingers with probability 1/3, and three fingers with probability 1/6. The expected outcome is that player A will win 2/3 point per round, on average. **43.** Write moves as (x, y) where x represents the number of regiments sent to the first location and y represents the number sent to the second location. Colonel Blotto should play (0, 4) with probability 4/9, (2, 2) with probability 1/9, and (4, 0) with probability 4/9. Captain Kije has several optimal strategies, one of which is to play (0, 3) with probability 1/30, (1, 2) with probability 8/15, (2, 1) with probability 16/45, and (3, 0) with probability 7/90. The expected outcome is that Colonel Blotto will win 14/9 points on average. **45.** The dual of a standard minimization problem satisfying the nonnegative objective condition is a standard maximization problem, which can be solved using the standard simplex algorithm, thus avoiding the need to do Phase I. **47.** Answers will vary. An example is: Minimize $c = x - y$ subject to $x - y \geq 100$, $x + y \geq 200$, $x \geq 0$, $y \geq 0$. This problem can be solved using the techniques in Section 4.4. **49.** Build 1 convention-style hotel, 4 vacation-style hotels and 2 small motels. **51.** Answers will vary.

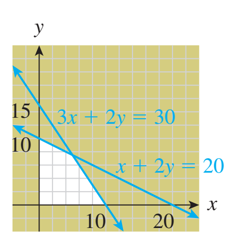
Chapter 4 Review

1.



Unbounded

3.



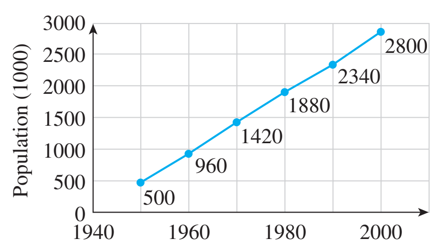
Bounded; Corner points: (0, 0), (0, 10), (5, 15/2), (10, 0)

5. $p = 21; x = 9, y = 3$ **7.** $c = 22; x = 8, y = 6$
9. $p = 45;\ x = 0, y = 15, z = 15$
11. $p = 220; x = 20, y = 20, z = 60$
13. $c = 30; x = 30, y = 0, z = 0$
15. $c = 50; x = 20, y = 10, z = 0, w = 20$
17. $c = 60; x = 24, y = 12$ **19.** $c = 20; x = 0, y = 20$
21. $R = [1/2 \quad 1/2 \quad 0], C = [0 \quad 1/3 \quad 2/3]^T, e = 0$
23. $R = [1/27 \quad 7/9 \quad 5/27], C = [8/27 \quad 5/27 \quad 14/27]^T$, $e = 8/27$ **25.** (A) **27.** 35 **29.** (B), (D) **31.** 450 packages from Duffin House, and 375 from Higgins Press for a minimum cost of $52,500. **33.** $c = 90{,}000; x = 0, y = 600, z = 0$ **35.** Billy Sean should take the following combination: Sciences: 24 credits, Fine Arts: no credits, Liberal Arts: 48 credits, Mathematics: 48 credits, for a total cost of $26,400. **37.** FantasyBooks should choose between "2 for 1" and "3 for 2" with probabilities 20% and 80%, respectively. O'HaganBooks should choose between "3 for 1" and "Finite Math" with probabilities 60% and 40%, respectively. O'HaganBooks expects to gain 12,000 customers from FantasyBooks.

Chapter 5

Section 5.1

1. $INT = \$120$, $FV = \$2120$ **3.** $INT = \$505$, $FV = \$20{,}705$ **5.** $INT = \$250$, $FV = \$10{,}250$ **7.** $PV = \$9090.91$ **9.** $PV = \$966.18$ **11.** $PV = \$14{,}457.83$ **13.** $5200 **15.** $787.40 **17.** 5% **19.** In 2 years **21.** 3.775% **23.** 65% **25.** 10% **27.** 168.85% **29.** 85.28% if you sold in February, 2005 **31.** No. Simple interest increase is linear. The graph is visibly not linear in that time period. **33.** 9.2% **35.** 3,260,000 **37.** $P = 500 + 46t$ (t = time in years since 1950) Graph:



39. Graph (A) is the only possible choice, because the equation $FV = PV(1 + rt) = PV + PVrt$ gives the future value as a linear function of time. **41.** Wrong. In simple interest growth, the change each year is a fixed percentage of the *starting* value, and not the preceding year's value. (Also see Exercise 42.) **43.** Simple interest is always calculated on a constant amount, PV. If interest is paid into your account, then the amount on which interest is calculated does not remain constant.

Section 5.2

1. $13,439.16 **3.** $11,327.08 **5.** $19,154.30 **7.** $12,709.44 **9.** $613.91 **11.** $810.65 **13.** $1227.74 **15.** 5.09% **17.** 10.47% **19.** 10.52% **21.** $268.99 **23.** $2491.75 **25.** $2927.15 **27.** $21,161.79 **29.** $163,414.56 **31.** $55,526.45 per year **33.** $174,110 **35.** $750.00 **37.** $27,171.92 **39.** $111,678.96 **41.** $1039.21 **43.** The one earning 11.9% compounded monthly **45.** Yes. The investment will have grown to about $150,281 million **47.** 147 reals **49.** 744 pesos **51.** 1224 pesos **53.** The Ecuadorian investment is better: it is worth 1.01614 units of currency (in constant units) per unit invested as opposed to 1.01262 units for Chile. **55.** 41.02% **57.** 51.90% if you sold in February, 2005 **59.** No. Compound interest increase is exponential. The graph looks roughly exponential in that period, but to really tell we can compare interest rates between marked points to see if the rate remained roughly constant: From December 1997 to August 1999 the rate was $(16.31/3.28)^{12/20} - 1 = 1.6179$ or 161.79%, while from August 1999 to March 2000 the rate was $(33.95/16.31)^{12/7} - 1 = 2.5140$ or 251.40%. These rates are quite different. **61.** 31 years; about $26,100 **63.** 2.3 years **65. a.** $1510.31 **b.** $54,701.29 **c.** 23.51% **67.** The function $y = P(1 + r/m)^{mx}$ is not a linear function of x, but an exponential function. Thus, its graph is not a straight line. **69.** Wrong. Its growth is exponential and can be modeled by $0.01(1.10)^t$. **71.** The graphs are the same because the formulas give the same function of x; a compound-interest investment behaves as though it was being compounded once a year at the effective rate. **73.** The effective rate exceeds the nominal rate when the interest is compounded more than once a year because then interest is being paid on interest accumulated during each year, resulting in a larger effective rate. Conversely, if the interest is compounded less often than once a year, the effective rate is less than the nominal rate. **75.** Compare their future values in constant dollars. The investment with the larger future value is the better investment. **77.** The graphs are approaching a particular curve as m gets larger, approximately the curve given by the largest two values of m.

Section 5.3

1. $15,528.23 **3.** $171,793.82 **5.** $23,763.28 **7.** $147.05 **9.** $491.12 **11.** $105.38 **13.** $90,155.46 **15.** $69,610.99 **17.** $95,647.68 **19.** $554.60 **21.** $1366.41 **23.** $524.14 **25.** $248.85 **27.** $1984.65 **29.** $999.61 **31.** $998.47 **33.** 3.617% **35.** 3.059% **37.** $973.54 **39.** $7451.49 **41.** You should take the loan from Solid Savings & Loan: it will have payments of $248.85 per month. The payments on the other loan would be more than $300 per month. **43.** Answers using correctly rounded intermediate results:

Year	Interest	Payment on Principal
1	$3934.98	$1798.98
2	$3785.69	$1948.27
3	$3623.97	$2109.99
4	$3448.84	$2285.12
5	$3259.19	$2474.77
6	$3053.77	$2680.19
7	$2831.32	$2902.64
8	$2590.39	$3143.57
9	$2329.48	$3404.48
10	$2046.91	$3687.05
11	$1740.88	$3993.08
12	$1409.47	$4324.49
13	$1050.54	$4683.42
14	$661.81	$5072.15
15	$240.84	$5491.80

45. First five years: $402.62/month; last 25 years: $601.73 **47.** Original monthly payments were $824.79. The new monthly payments will be $613.46. You will save $36,481.77 in interest. **49.** 10.81% **51.** 13 years **53.** 4.5 years **55.** 24 years **57.** He is wrong because his estimate ignores the interest that will be earned by your annuity—both while it is increasing and while it is decreasing. Your payments will be considerably smaller (depending on the interest earned). **59.** He is not correct. For instance, the payments on a $100,000 10-year mortgage at 12% are $1434.71, while for a 20-year mortgage at the same rate, they are $1101.09, which is a lot more than half the 10-year mortgage payment. **61.** $PV = FV(1+i)^{-n} =$

$$PMT\frac{(1+i)^n - 1}{i}(1+i)^{-n} = PMT\frac{1-(1+i)^{-n}}{i}$$

Chapter 5 Review

1. $7425.00 **3.** $7604.88 **5.** $6757.41 **7.** $4848.48 **9.** $4733.80 **11.** $5331.37 **13.** $177.58 **15.** $112.54 **17.** $187.57 **19.** $9584.17 **21.** 5.346% **23.** 14.0 years **25.** 10.8 years **27.** 7.0 years **29.** 2003

Year	2000	2001	2002	2003	2004
Revenue	$180,000	$216,000	$259,200	$311,040	$373,248

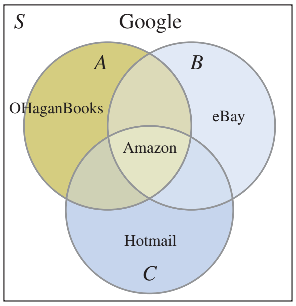
31. At least 52,515 shares **33.** $224,111 **35.** $420,275 **37.** $1453.06 **39.** $53,055.66 **41.** 5.99%

Chapter 6

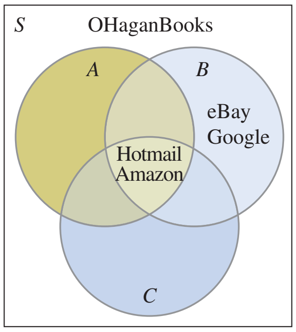
Section 6.1

1. F = {spring, summer, fall, winter} **3.** $I = \{1, 2, 3, 4, 5, 6\}$ **5.** $A = \{1, 2, 3\}$ **7.** $B = \{2, 4, 6, 8\}$ **9. a.** S = {(H, H), (H, T), (T, H), (T, T)} **b.** S = {(H, H), (H, T), (T, T)} **11.** $S = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$ **13.** $S = \{(1, 5), (2, 4), (3, 3)\}$ **15.** $S = \emptyset$

17.

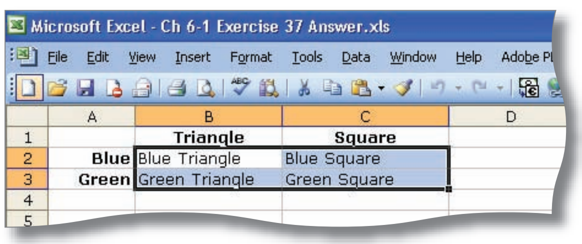


19.



21. A **23.** A **25.** {June, Janet, Jill, Justin, Jeffrey, Jello, Sally, Solly, Molly, Jolly} **27.** {Jello} **29.** $\emptyset$ **31.** {Jello} **33.** {Janet, Justin, Jello, Sally, Solly, Molly, Jolly} **35.** {(small, triangle), (small, square), (medium, triangle), (medium, square), (large, triangle), (large, square)} **37.** {(small, blue), (small, green), (medium, blue), (medium, green), (large, blue), (large, green)}

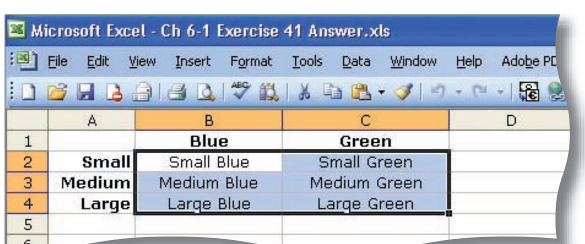
39.



Microsoft Excel - Ch 6-1 Exercise 37 Answer.xls

	A	B	C	D
1		Triangle	Square	
2	Blue	Blue Triangle	Blue Square	
3	Green	Green Triangle	Green Square	
4				
5				

41.

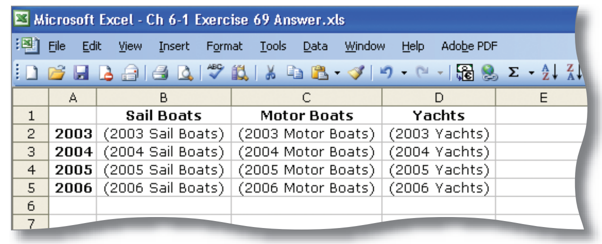


Microsoft Excel - Ch 6-1 Exercise 41 Answer.xls

	A	B	C	D
1		Blue	Green	
2	Small	Small Blue	Small Green	
3	Medium	Medium Blue	Medium Green	
4	Large	Large Blue	Large Green	
5				
6				

43. $B \times A =$ {1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T} **45.** $A \times A \times A =$ {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} **47.** {(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)} **49.** $\emptyset$ **51.** {(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5), (2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)} **61.** $A \cap B =$ {Acme, Crafts} **63.** $B \cup C =$ {Acme, Brothers, Crafts, Dion, Effigy, Global, Hilbert} **65.** $A' \cap C =$ {Dion, Hilbert} **67.** $A \cap B' \cap C' = \emptyset$

69.



Microsoft Excel - Ch 6-1 Exercise 69 Answer.xls

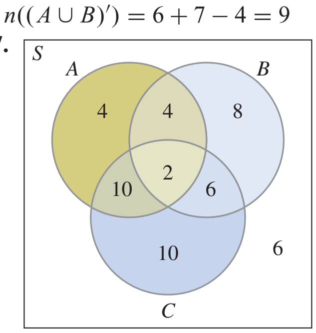
	A	B	C	D	E
1		Sail Boats	Motor Boats	Yachts	
2	2003	(2003 Sail Boats)	(2003 Motor Boats)	(2003 Yachts)	
3	2004	(2004 Sail Boats)	(2004 Motor Boats)	(2004 Yachts)	
4	2005	(2005 Sail Boats)	(2005 Motor Boats)	(2005 Yachts)	
5	2006	(2006 Sail Boats)	(2006 Motor Boats)	(2006 Yachts)	
6					
7					

71. $I \cup J$ **73.** (B) **75.** Answers may vary. Let $A = \{1\}$, $B = \{2\}$, and $C = \{1, 2\}$. Then $(A \cap B) \cup C = \{1, 2\}$ but $A \cap (B \cup C) = \{1\}$. In general, $A \cap (B \cup C)$ must be a subset of A, but $(A \cap B) \cup C$ need not be; also, $(A \cap B) \cup C$ must contain C as a subset, but $A \cap (B \cup C)$ need not. **77.** A universal set is a set containing all "things" currently under consideration. When discussing sets of positive integers, the universe might be the set of all positive integers, or the set of all integers (positive, negative, and 0), or any other set containing the set of all positive integers. **79.** A is the set of suppliers who deliver components on time, B is the set of suppliers whose components are known to be of high quality, and C is the set of suppliers who do not promptly replace defective components. **81.** Let $A =$ {movies that are violent}, $B =$ {movies that are shorter than two hours}, $C =$ {movies that have a tragic ending}, and $D =$ {movies that have an unexpected ending}. The given sentence can be rewritten as "She prefers movies in $A' \cap B \cap (C \cup D)'$." It can also be rewritten as "She prefers movies in $A' \cap B \cap C' \cap D'$."

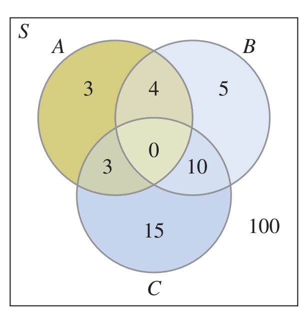
Section 6.2

1. 9 **3.** 7 **5.** 4 **7.** $n(A \cup B) = 7, n(A) + n(B) - n(A \cap B) = 4 + 5 - 2 = 7$ **9.** 4 **11.** 18 **13.** 72 **15.** 60 **17.** 20 **19.** 6 **21.** 9 **23.** 4 **25.** $n((A \cap B)') = 9, n(A') + n(B') - n((A \cup B)') = 6 + 7 - 4 = 9$

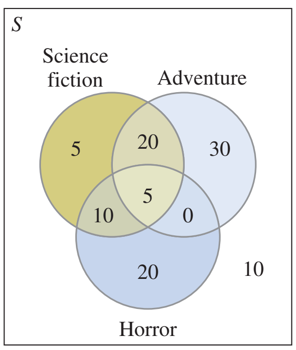
27.



29.



31. 76,000 **33.** 2 **35.** $C \cap N$ is the set of authors who are both successful and new. $C \cup N$ is the set of authors who are either successful or new (or both). $n(C) = 30$; $n(N) = 20$; $n(C \cap N) = 5$; $n(C \cup N) = 45$; $45 = 30 + 20 - 5$
37. $C \cap N'$ is the set of authors who are successful but not new. $n(C \cap N') = 25$ **39.** 31.25%; 83.33% **41.** $N \cap C$; $n(N \cap C) = 8$ billion **43.** $C \cap N'$; $n(C \cap N') = 13$ billion
45. $A \cap (N \cup U)$; $n(A \cap (N \cup U)) = 14$ billion
47. $V \cap I'$; $n(V \cap I') = 15$ **49.** 80; The number of stocks that were either not pharmaceutical stocks, or were unchanged in value after a year (or both). **51.** 3/8; the fraction of Internet stocks that increased in value **53. a.** 931 **b.** 382
55. a.



b. 37.5% **57.** 17 **59.** The number of elements in the Cartesian product of two finite sets is the product of the number of elements in the two sets. **61.** Answers will vary.
63. When $A \cap B \neq \emptyset$ **65.** When $B \subseteq A$
67. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Section 6.3

1. 10 **3.** 30 **5.** 6 outcomes **7.** 15 outcomes
9. 13 outcomes **11.** 25 outcomes **13.** 4 **15.** 93
17. 16 **19.** 30 **21.** 13 **23.** 18 **25.** 25,600 **27.** 3381
29. a. 288 **b.** 288 **31.** 256 **33.** 10 **35.** 286 **37.** 4
39. a. 8,000,000 **b.** 30,000 **c.** 4,251,528 **41. a.** $4^3 = 64$
b. 4^n **c.** $4^{2.1\times10^{10}}$ **43. a.** $16^6 = 16{,}777{,}216$
b. $16^3 = 4096$ **c.** $16^2 = 256$ **d.** 766
45. $(10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4) \times (8 \times 7 \times 6 \times 5)$ $= 1{,}016{,}064{,}000$ possible casts **47. a.** $26^3 \times 10^3 = 17{,}576{,}000$
b. $26^2 \times 23 \times 10^3 = 15{,}548{,}000$ **c.** $15{,}548{,}000 - 3 \times 10^3 = 15{,}545{,}000$ **49. a.** 4 **b.** 4 **c.** There would be an infinite number of routes. **51. a.** 72 **b.** 36 **53.** 96 **55. a.** 36
b. 37 **57.** Step 1: Choose a day of the week on which Jan 1 will fall: 7 choices. Step 2: Decide whether or not it is a leap year: 2 choices. Total: $7 \times 2 = 14$ possible calendars. **59.** 1900
61. Step 1: choose a position in the Left-Right direction: m choices. Step 2: choose a position in the Front-Back direction: n choices. Step 3: choose a position in the Up-Down direction: r choices. Hence there are $m \cdot n \cdot r$ possible outcomes. **63.** 4 **65.** Cartesian product
67. The decision algorithm produces every pair of shirts twice, first in one order and then in the other. **69.** Think of placing the five squares in a row of five empty slots. Step 1: choose a slot for the blue square, 5 choices. Step 2: choose a slot for the green square, 4 choices. Step 3: choose the remaining 3 slots for the yellow squares, 1 choice. Hence there are 20 possible five-square sequences.

Section 6.4

1. 720 **3.** 56 **5.** 360 **7.** 15 **9.** 3 **11.** 45 **13.** 20
15. 4950 **17.** 360 **19.** 35 **21.** 120 **23.** 120 **25.** 20
27. 60 **29.** 210 **31.** 7 **33.** 35 **35.** 24 **37.** 126
39. 196 **41.** 105 **43.** $\dfrac{C(30, 5) \times 5^{25}}{6^{30}} \approx 0.192$
45. $\dfrac{C(30, 15) \times 3^{15} \times 3^{15}}{6^{30}} \approx 0.144$ **47.** 24
49. $C(13, 2)C(4, 2)C(4, 2) \times 44 = 123{,}552$
51. $13 \times C(4, 2)C(12, 3) \times 4 \times 4 \times 4 = 1{,}098{,}240$
53. 10,200 **55. a.** 252 **b.** 20 **c.** 26 **57. a.** 300 **b.** 3 **c.** 1 in 100 or .01 **59. a.** 210 **b.** 77 **c.** No **61. a.** 23! **b.** 18!
c. $19 \times 18!$ **63.** $C(11, 1)C(10, 4)C(6, 4)C(2, 2)$
65. $C(11, 2)C(9, 1)C(8, 1)C(7, 3)C(4, 1)C(3, 1)C(2, 1)C(1, 1)$
67. $C(10, 2)C(8, 4)C(4, 1)C(3, 1)C(2, 1)C(1, 1)$
69. (A) **71.** (D) **73. a.** 9880 **b.** 1560 **c.** 11,480
75. a. $C(20, 2) = 190$ **b.** $C(n, 2)$ **77.** The multiplication principle; it can be used to solve all problems that use the formulas for permutations. **79.** Urge your friend not to focus on formulas, but instead learn to formulate decision algorithms and use the principles of counting. **81.** It is ambiguous on the following point: are the three students to play different characters, or are they to play a group of three, such as "three guards." This should be made clear in the exercise.

Chapter 6 Review

1. $N = \{-3, -2, -1\}$ **3.** $S = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)\}$ **5.** $A \cup B' = \{a, b, d\}$, $A \times B' = \{(a, a), (a, d), (b, a), (b, d)\}$ **7.** $A \times B$
9. $(P \cap E' \cap Q)'$ or $P' \cup E \cup Q'$ **11.** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, $n(C') = n(S) - n(C)$; 100
13. $n(A \times B) = n(A)n(B)$, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, $n(A') = n(S) - n(A)$, 21
15. $C(12, 1)C(4, 2)C(11, 3)\ C(4, 1)C(4, 1)C(4, 1)$
17. $C(4, 1)C(10, 1)$ **19.** $C(4, 4)C(8, 1) = 8$
21. $C(3, 2)C(9, 3) + C(3, 3)C(9, 2) = 288$ **23.** The set of books that are either sci-fi or stored in Texas (or both); $n(S \cup T) = 112{,}000$ **25.** The set of books that are either stored in California or not sci-fi; $n(C \cup S') = 175{,}000$
27. The romance books that are also horror books or stored in Texas; $n(R \cap (T \cup H)) = 20{,}000$ **29.** 1000
31. FarmerBooks.com; 1800 **33.** JungleBooks.com; 3500
35. 15,600 **37.** 2 letters, 4 digits; 2,948,400 **39.** 28,000

Chapter 7

Section 7.1

1. $S = \{HH, HT, TH, TT\}$; $E = \{HH, HT, TH\}$ **3.** $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$; $E = \{HTT, THT, TTH, TTT\}$

5. $S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\}$;

$E = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

7. $S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ & & (3,3) & (3,4) & (3,5) & (3,6) \\ & & & (4,4) & (4,5) & (4,6) \\ & & & & (5,5) & (5,6) \\ & & & & & (6,6) \end{array} \right\}$;

$E = \{(1,3), (2,2)\}$

9. S as in Exercise 7; $E = \{(2, 2), (2, 3), (2, 5), (3, 3), (3, 5), (5, 5)\}$ **11.** $S = \{$m, o, z, a, r, t$\}$: $E = \{$o, a$\}$ **13.** $S = \{$(s, o), (s, r), (s, e), (o, s), (o, r), (o, e), (r, s), (r, o), (r, e), (e, s), (e, o), (e, r)$\}$; $E = \{$(o, s), (o, r), (o, e), (e, s), (e, o), (e, r)$\}$ **15.** $S = \{01, 02, 03, 04, 10, 12, 13, 14, 20, 21, 23, 24, 30, 31, 32, 34, 40, 41, 42, 43\}$; $E = \{10, 20, 21, 30, 31, 32, 40, 41, 42, 43\}$ **17.** $S = \{$domestic car, imported car, van, antique car, antique truck$\}$; $E = \{$van, antique truck$\}$ **19. a.** all sets of 4 gummy candies chosen from the packet of 12 **b.** all sets of 4 gummy candies in which two are strawberry and two are blackcurrant **21. a.** all lists of 14 people chosen from 20 **b.** all lists of 14 people chosen from 20, in which Colin Powell occupies the first position **23.** $A \cap B$; $n(A \cap B) = 1$ **25.** B'; $n(B') = 33$ **27.** $B' \cap D'$; $n(B' \cap D') = 2$ **29.** $C \cup B$; $n(C \cup B) = 12$ **31.** $W \cap I$ **33.** $E \cup I'$ **35.** $I \cup (W \cap E')$ **37.** $(I \cup W) \cap E'$ **39.** $E = \{$New England, Pacific, Middle Atlantic$\}$ **41.** $E \cup F$ is the event that you choose a region that saw an increase in housing prices of 15% or more or is on the east coast. $E \cup F = \{$Pacific, New England, Middle Atlantic, South Atlantic$\}$. $E \cap F$ is the event that you choose a region that saw an increase in housing prices of 15% or more and is on the east coast. $E \cap F = \{$New England, Middle Atlantic$\}$. **43. a.** Mutually exclusive **b.** Not mutually exclusive **45.** $S \cap N$ is the event that an author is successful and new. $S \cup N$ is the event that an author is either successful or new; $n(S \cap N) = 5$; $n(S \cup N) = 45$ **47.** N and E **49.** $S \cap N'$ is the event that an author is successful but not a new author. $n(S \cap N') = 25$ **51.** 31.25%; 83.33% **53.** $V \cap I'$; $n(V \cap I') = 15$ **55.** 80; The number of stocks that were either not pharmaceutical stocks, or were unchanged in value after a year (or both). **57.** P and E, P and I, E and I, N and E, V and N, V and D, N and D **59.** 3/8; the fraction of Internet stocks that increased in value **61. a.** $E' \cap H$ **b.** $E \cup H$ **c.** $(E \cup G)' = E' \cap G'$ **63. a.** $\{9\}$ **b.** $\{6\}$ **65. a.** The dog's "fight" drive is weakest. **b.** The dog's "fight" and "flight" drives are either both strongest or both weakest. **c.** Either the dog's "fight" drive is strongest, or its "flight" drive is strongest. **67.** $C(6, 4) = 15$; $C(1, 1)C(5, 3) = 10$ **69. a.** $n(S) = P(7, 3) = 210$ **b.** $E \cap F$ is the event that Celera wins and Electoral College is in second or third place. In other words, it is the set of all lists of three horses in which Celera is first and Electoral College is second or third. $n(E \cap F) = 10$. **71.** $C(8, 3) = 56$ **73.** $C(4, 1)\,C(2, 1)\,C(2, 1) = 16$ **75.** Subset of the sample space **77.** E and F do not both occur **79.** True; Consider the following experiment: Select an element of the set S at random. **81.** Answers may vary. Cast a die and record the remainder when the number facing up is divided by 2. **83.** Yes. For instance, $E = \{(2, 5), (5, 1)\}$ and $F = \{(4, 3)\}$ are two such events.

Section 7.2

1. .4 **3.** .8

5.

Outcome	HH	HT	TH	TT
Probability	.275	.2375	.3	.1875

7. .575 **9.** The second coin *seems* slightly biased in favor of heads, because heads comes up approximately 58% of the time. On the other hand, it is conceivable that the coin is fair and that heads came up 58% of the time purely by chance. Deciding which conclusion is more reasonable requires some knowledge of inferential statistics. **15.** $P(E) = 1/4$ **17.** $P(E) = 1$ **19.** $P(E) = 3/4$ **21.** $P(E) = 3/4$ **23.** $P(E) = 1/2$ **25.** $P(E) = 1/9$ **27.** $P(E) = 0$ **29.** $P(E) = 1/4$ **31.** 1/12; $\{(4, 4), (2, 3), (3, 2)\}$

33.

Outcome	1	2	3	4	5	6
Probability	1/9	2/9	1/9	2/9	1/9	2/9

$P(\{1, 2, 3\}) = 4/9$

35.

Outcome	1	2	3	4
Probability	8/15	4/15	2/15	1/15

37. a. .04 **b.** .98

39. a.

Test Rating	3	2	1	0
Probability	.1	.4	.4	.1

b. 0.5 **41. a.** Dial-up: .63, Cable Modem: .21, DSL: .15, Other: .01 **b.** .36

43.

Outcome	Low	Middle	High
Probability	.5	.3	.2

45. .25 **47.** .2 **49.** .7 **51.** 5/6 **53.** 5/16

55.

Outcome	U	C	R
Probability	.2	.64	.16

57.

Conventional	No pesticide	Single pesticide	Multiple pesticide
Probability	.27	.13	.60
Organic	No pesticide	Single pesticide	Multiple pesticide
Probability	.77	.13	.10

59. $P(\text{false negative}) = 10/400 = .025$, $P(\text{false positive}) = 10/200 = .05$ **63.** .70 **65.** .86 **67.** .86
69.

Outcome	Hispanic or Latino	White (not Hispanic)	African American	Asian	Other
Probability	.42	.37	.09	.08	.04

$P(\text{Neither White nor Asian}) = .55$
71. a. $S =$ {Stock market success, Sold to other concern, Fail}

b.

Outcome	Stock market success	Sold to other concern	Fail
Probability	.2	.3	.5

c. .5

73.

Outcome	SUV	Pickup	Passenger Car	Minivan
Probability	.25	.15	.50	.10

75. $P(1) = 0$, $P(6) = 0$; $P(2) = P(3) = P(4) = P(5) = 1/4 = .25$ **77.** $P(1) = P(6) = 1/10$; $P(2) = P(3) = P(4) = P(5) = 1/5$, $P(\text{odd}) = 1/2$ **79.** $P(1,1) = P(2,2) = \ldots = P(6,6) = 1/66$; $P(1,2) = \ldots = P(6,5) = 1/33$, $P(\text{odd sum}) = 6/11$ **81.** $P(2) = 15/38$; $P(4) = 3/38$, $P(1) = P(3) = P(5) = P(6) = 5/38$, $P(\text{odd}) = 15/38$
83. The fraction of times E occurs **85.** Wrong. For a pair of fair dice, the theoretical probability of a pair of matching numbers is $1/6$, as Ruth says. However, it is quite possible, although not very likely, that if you cast a pair of fair dice 20 times, you will never obtain a matching pair (in fact, there is approximately a 2.6% chance that this will happen). In general, a nontrivial claim about theoretical probability can never be absolutely validated or refuted experimentally. All we can say is that the evidence suggests that the dice are not fair. **87.** For a (large) number of days, record the temperature prediction for the next day and then check the actual temperature the next day. Record whether the prediction was accurate (within, say, 2°F of the actual temperature). The fraction of times the prediction was accurate is the estimated probability. **89.** He is wrong. It is possible to have a run of losses of any length. Tony may have grounds to *suspect* that the game is rigged, but no proof.

Section 7.3

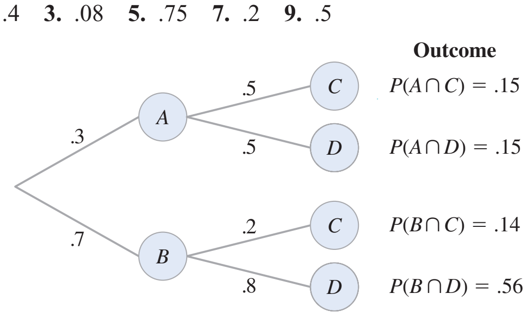
1. .65 **3.** .1 **5.** .7 **7.** .4 **9.** .25 **11.** 1.0 **13.** .3
15. 1.0 **17.** No; $P(A \cup B)$ should be $\le P(A) + P(B)$.
19. Yes **21.** No; $P(A \cup B)$ should be $\ge P(B)$.
23. $P(e) = .2$ **a.** .9 **b.** .95 **c.** .1 **d.** .8 **25.** 5/6
27. .39 **29.** .54 **31.** .24 **33.** .00 **35.** .76 **37.** .46
39. .54 **41.** .01 **43.** .56 **45.** .43 **47.** 22%; 100%
49. All of them **51.** .884 **53.** They are mutually exclusive.
55. Wrong. For example, the theoretical probability of winning a state lotto is small but nonzero. However, the vast majority of people who play lotto day of their lives never win, no matter how frequently they play. **57.** When $A \cap B = \emptyset$ we have $P(A \cap B) = P(\emptyset) = 0$, so $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - 0 = P(A) + P(B)$. **59.** Zero. According to the assumption, no matter how many thunderstorms occur, lightning cannot strike your favorite spot more than once, and so, after n trials the estimated probability will never exceed $1/n$, and so will approach zero as the number of trials gets large.
61. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Section 7.4

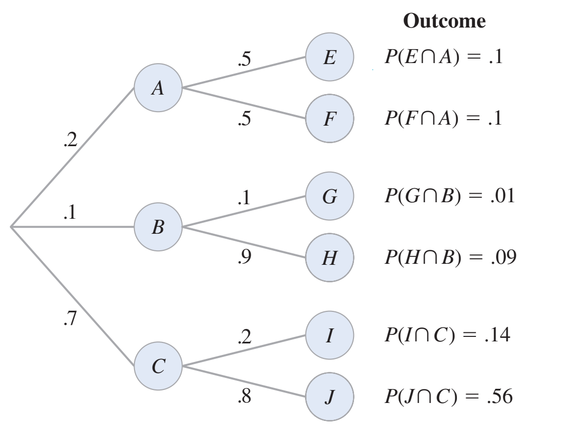
1. 1/42 **3.** 7/9 **5.** 1/7 **7.** 1/2 **9.** 41/42 **11.** 1/15
13. 4/15 **15.** 1/5 **17.** $1/(2^8 \times 5^5 \times 5!)$ **19.** .4226
21. .0475 **23.** .0020 **25.** $1/27^{39}$ **27.** 1/7 **29.** Probability of being a big winner $= 1/2{,}118{,}760 \approx .000000472$. Probability of being a small-fry winner $= 225/2{,}118{,}760 \approx .000106194$. Probability of being either a winner or a small-fry winner $= 226/2{,}118{,}760 \approx .000106666$.
31. a. $C(600,300)/C(700,400)$ **b.** $C(699,399)/C(700,400)$ or 400/700 **33.** $P(10,3)/10^3 = 18/25 = .72$ **35.** $8!/8^8$
37. 1/8 **39.** 1/8 **41.** 37/10,000 **43. a.** 90,720 **b.** 25,200
c. $25{,}200/90{,}720 = 25/90 \approx .28$ **45.** The four outcomes listed are not equally likely; for example, (red, blue) can occur in four ways. The methods of this section yield a probability for (red, blue) of $C(2,2)/C(4,2) = 1/6$ **47.** No. If we do not pay attention to order, the probability is $C(5,2)/C(9,2) = 10/36 = 5/18$. If we do pay attention to order, the probability is $P(5,2)/P(9,2) = 20/72 = 5/18$ again. The difference between permutations and combinations cancels when we compute the probability. **49.** Answers will vary.

Section 7.5

1. .4 **3.** .08 **5.** .75 **7.** .2 **9.** .5
11.

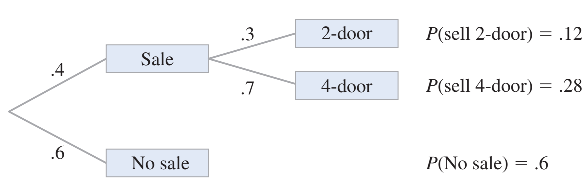


13.

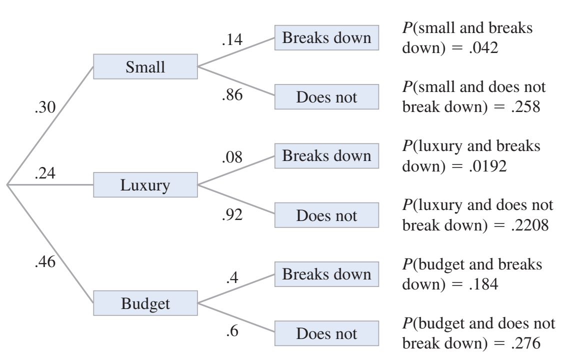


15. 1/10 **17.** 1/5 **19.** 2/9 **21.** 1/84 **23.** 5/21 **25.** 24/175 **27.** (B) **29.** (C) **31.** $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ Independent **33.** $\frac{5}{18} \cdot \frac{1}{2} \neq \frac{1}{9}$ Dependent **35.** $\frac{25}{36} \cdot \frac{5}{18} \neq \frac{2}{9}$ Dependent **37.** $(1/2)^{11} = 1/2048$ **39.** .8 **41.** .43 **43.** .34 **45.** Not independent; P(giving up | used Brand X) = 0.1 is larger than P(giving up) **47.** .00015 **49.** 5/6 **51.** 3/4 **53.** 11/16 **55.** 11/14

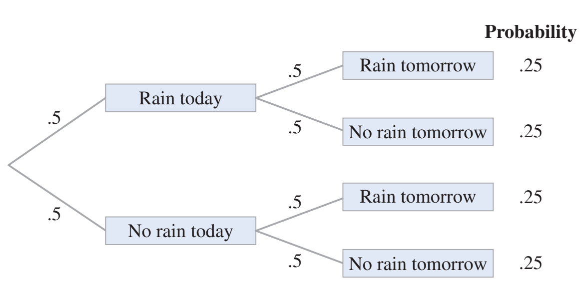
57.



59.



61.



63. .76 **65.** .33 **67.** .37 **69.** .98 **71.** The claim is correct. The probability that an unemployed person has a high school diploma only is .35, while the corresponding figure for an employed person is .30. **73.** $P(K \mid D) = 1.31P(K \mid D')$ **75. a.** $P(I \mid T) > P(I)$ **b.** It was ineffective. **77. a.** .59 **b.** \$35,000 or more: P(Internet user | < \$35,000) ≈ .27 < P(Internet user | ≥ \$35,000) ≈ .59. **79.** $P(R \mid J)$ **81.** (D) **83. a.** .000057 **b.** .015043 **85.** 11% **87.** 106 **89.** .631 **91.** Answers will vary. Here is a simple one: E: the first toss is a head, F: the second toss is a head, G: the third toss is a head. **93.** The probability you seek is $P(E \mid F)$, or should be. If, for example, you were going to place a wager on whether E occurs or not, it is crucial to know that the sample space has been reduced to F (you know that F did occur). If you base your wager on $P(E)$ rather than $P(E \mid F)$ you will misjudge your likelihood of winning. **95.** If $A \subseteq B$ then $A \cap B = A$, so $P(A \cap B) = P(A)$ and $P(A \mid B) = P(A \cap B)/P(B) = P(A)/P(B)$. **97.** Your friend is correct. If A and B are mutually exclusive then $P(A \cap B) = 0$. On the other hand, if A and B are independent then $P(A \cap B) = P(A)P(B)$. Thus, $P(A)P(B) = 0$. If a product is 0 then one of the factors must be 0, so either $P(A) = 0$ or $P(B) = 0$. Thus, it cannot be true that A and B are mutually exclusive, have nonzero probabilities, and are independent all at the same time. **99.** $P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] = 1 - [P(A) + P(B) - P(A)P(B)] = (1 - P(A))(1 - P(B)) = P(A')P(B')$.

Section 7.6

1. .4 **3.** .7887 **5.** .7442 **7.** .1163 **9.** 26.8% **11.** .1724 **13.** .73 **15.** .71 **17.** .1653 **19.** 88% **21.** 12% **23. a.** 14.43%; **b.** 19.81% of single homeowners have pools. Thus they should go after the single homeowners. **25.** 9 **27.** .9310 **29.** 1.76% **31.** .20 **33.** K: child killed; D: Airbag deployed; $P(K \mid D) = 1.31P(K \mid D')$; $P(D \mid K) = 1.31(.25)/[1.31(.25) + .75] = .30$ **35.** Show him an example like Example 1 of this section, where $P(T \mid A) = .95$ but $P(A \mid T) \approx .64$. **37.** Suppose that the steroid test gives 10% false negatives and that only 0.1% of the tested population uses steroids. Then the probability that an athlete uses steroids, given that he or she has tested positive, is

$$\frac{(.9)(.001)}{(.9)(.001) + (.01)(.999)} \approx .083.$$

39. Draw a tree in which the first branching shows which of R_1, R_2, or R_3 occurred, and the second branching shows which of T or T' then occurred. There are three final outcomes in which T occurs: $P(R_1 \cap T) = P(T \mid R_1)\ P(R_1)$, $P(R_2 \cap T) = P(T \mid R_2)P(R_2)$, and $P(R_3 \cap T) = P(T \mid R_3)P(R_3)$. In only one of these, the first, does R_1 occur. Thus,

$$P(R_1 \mid T) = \frac{P(R_1 \cap T)}{P(T)}$$

$$= \frac{P(T \mid R_1)P(R_1)}{P(T \mid R_1)P(R_1) + P(T \mid R_2)P(R_2) + P(T \mid R_3)P(R_3)}$$

41. The reasoning is flawed. Let A be the event that a Democrat agrees with Safire's column, and let F and M be the events that a Democrat reader is female and male respectively. Then A. D. makes the following argument:

$P(M \mid A) = 0.9$, $P(F \mid A') = 0.9$. Therefore, $P(A \mid M) = 0.9$.

According to Bayes' Theorem we cannot conclude anything about $P(A \mid M)$ unless we know $P(A)$, the percentage of all Democrats who agreed with Safire's column. This was not given.

Section 7.7

1. $\begin{bmatrix} 1/4 & 3/4 \\ 1/2 & 1/2 \end{bmatrix}$ **3.** $\begin{bmatrix} 0 & 1 \\ 1/6 & 5/6 \end{bmatrix}$ **5.** $\begin{bmatrix} 0 & .8 & .2 \\ .9 & 0 & .1 \\ 0 & 0 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **9.** $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2/3 & 0 & 1/3 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

11. a. $\begin{bmatrix} .25 & .75 \\ 0 & 1 \end{bmatrix}$ **b.** distribution after one step: [.5 .5]; after two steps: [.25 .75]; after three steps: [.125 .875]

13. a. $\begin{bmatrix} .36 & .64 \\ .32 & .68 \end{bmatrix}$ **b.** distribution after one step: [.3 .7]; after two steps: [.34 .66]; after three steps: [.332 .668]

15. a. $\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$ **b.** distribution after one step: [2/3 1/3]; after two steps: [2/3 1/3]; after three steps: [2/3 1/3]

17. a. $\begin{bmatrix} 3/4 & 1/4 \\ 3/4 & 1/4 \end{bmatrix}$ **b.** distribution after one step: [3/4 1/4]; after two steps: [3/4 1/4]; after three steps: [3/4 1/4]

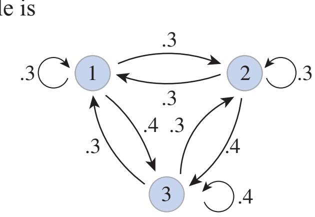
19. a. $\begin{bmatrix} .25 & .75 & 0 \\ 0 & 1 & 0 \\ 0 & .75 & .25 \end{bmatrix}$ **b.** distribution after one step: [.5 .5 0]; after two steps: [.25 .75 0]; after three steps: [.125 .875 0] **21. a.** $\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 4/9 & 4/9 & 1/9 \\ 0 & 1 & 0 \end{bmatrix}$ **b.** distribution after one step: [1/2 1/2 0]; after two steps: [1/6 2/3 1/6]; after three steps: [7/18 7/18 2/9]

23. a. $\begin{bmatrix} .01 & .99 & 0 \\ 0 & 1 & 0 \\ 0 & .36 & .64 \end{bmatrix}$ **b.** distribution after one step: [.05 .55 .4]; after two steps: [.005 .675 .32]; after three steps: [.0005 .7435 .256] **25.** [2/3 1/3] **27.** [3/7 4/7] **29.** [2/5 3/5] **31.** [2/5 1/5 2/5] **33.** [1/3 1/2 1/6] **35.** [0 1 0] **37.** 1 = Sorey state, 2 = C&T; $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$; 3/8 = .375 **39. a.** 1 = not checked in; 2 = checked in $P = \begin{bmatrix} .4 & .6 \\ 0 & 1 \end{bmatrix}$, $P^2 = \begin{bmatrix} .16 & .84 \\ 0 & 1 \end{bmatrix}$, $P^3 = \begin{bmatrix} .064 & .936 \\ 0 & 1 \end{bmatrix}$ **b.** 1 hour: .6; 2 hours: .84; 3 hours: .936 **c.** Eventually, all the roaches will have checked in. **41.** 16.67% fall into the high-risk category and 83.33% into the low-risk category. **43. a.** $47/300 \approx .156667$ **b.** 3/13 **45.** 41.67% of the customers will be in the Paid up category, 41.67% in the 0–90 days category, and 16.67% in the bad debt category.

47. a. $P = \begin{bmatrix} .729 & .271 & 0 \\ .075 & .84 & .085 \\ 0 & .304 & .696 \end{bmatrix}$ **b.** 2.3%
c. Affluent: 17.8%; Middle class: 64.3%; Poor: 18.0%
49. Long-term income distribution (top to bottom): [8.43%, 41.57%, 41.57%, 8.43%]

51. a. $P = \begin{bmatrix} .981 & .005 & .005 & .009 \\ .01 & .972 & .006 & .012 \\ .01 & .006 & .973 & .011 \\ .008 & .006 & .005 & .981 \end{bmatrix}$

b. Verizon: 29.6%, Cingular: 19.3%, AT&T: 18.1%, Other: 32.8%
c. Verizon: 30.3%, Cingular: 18.6%, AT&T: 17.6%, Other: 33.5%. The biggest gainers are Verizon and Other, each gaining 0.6%. **53.** [1/5 1/5 1/5 1/5 1/5]
55. Answers will vary. **57.** There are two assumptions made by Markov systems that may not be true about the stock market: the assumption that the transition probabilities do not change over time, and the assumption that the transition probability depends only on the current state. **59.** If q is a row of Q, then by assumption $qP = q$. Thus, when we multiply the rows of Q by P, nothing changes, and $QP = Q$. **61.** At each step, only 0.4 of the population in state 1 remains there, and nothing enters from any other state. Thus, when the first entry in the steady-state distribution vector is multiplied by 0.4 it must remain unchanged. The only number for which this true is 0.
63. An example is



65. If $vP = v$ and $wP = w$, then $\frac{1}{2}(v + w)P = \frac{1}{2}vP + \frac{1}{2}wP = \frac{1}{2}v + \frac{1}{2}w = \frac{1}{2}(v + w)$. Further, if the entries of v and w add up to 1, then so do the entries of $(v + w)/2$.

Chapter 7 Review

1. $n(S) = 8$, $E = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$, $P(E) = 7/8$ **3.** $n(S) = 36$; $E = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$; $P(E) = 1/6$ **5.** $n(S) = 6$; $E = \{2\}$; $P(E) = 1/8$ **7.** .76 **9.** .25 **11.** .5 **13.** 7/15 **15.** 8/792 **17.** 48/792 **19.** 288/792 **21.** $C(8, 5)/C(52, 5)$
23. $C(4, 3)C(1, 1)C(3, 1)/C(52, 5)$
25. $C(9, 1)C(8, 1)C(4, 3)C(4, 2)/C(52, 5)$ **27.** 1/5; dependent **29.** 1/6; independent **31.** 1; dependent

33. $P = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$

35. Brand A: $65/192 \approx .339$, Brand B: $127/192 \approx .661$
37. 14/25 **39.** 15/94 **41.** 79/167 **43.** 98%
45. .931 **47.** 0.75% **49.** .0049 **51.** 40% for OHaganBooks.com, 26% for JungleBooks.com, and 34% for FarmerBooks.com **53.** Here are three: (1) it is possible for someone to be a customer at two different enterprises; (2) some customers may stop using all three of the companies; (3) new customers can enter the field.

Chapter 8

Section 8.1

1. Finite; {2, 3, . . . , 12} **3.** Discrete infinite; {0, 1, −1, 2, −2, . . .} (negative profits indicate loss) **5.** Continuous; X can assume any value between 0 and 60. **7.** Finite; {0, 1, 2, . . . , 10} **9.** Discrete infinite $\{k/1, k/4, k/9, k/16, \ldots\}$
11. a. $S = \{HH, HT, TH, TT\}$
b. X is the rule that assigns to each outcome the number of tails.
c.

Outcome	HH	HT	TH	TT
Value of X	0	1	1	2

13. a. $S = \{(1, 1), (1, 2), \ldots, (1, 6), (2, 1), (2, 2), \ldots, (6, 6)\}$
b. X is the rule that assigns to each outcome the sum of the two numbers.
c.

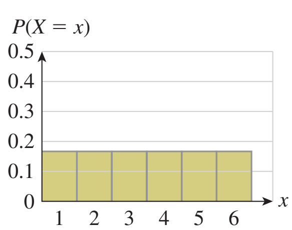
Outcome	(1, 1)	(1, 2)	(1, 3)	...	(6, 6)
Value of X	2	3	4	...	12

15. a. $S = \{(4, 0), (3, 1), (2, 2)\}$ (listed in order (red, green))
b. X is the rule that assigns to each outcome the number of red marbles.
c.

Outcome	(4, 0)	(3, 1)	(2, 2)
Value of X	4	3	2

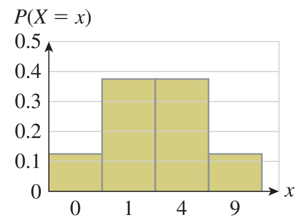
17. a. $S =$ the set of students in the study group
b. X is the rule that assigns to each student his or her final exam score. **c.** The values of X, in the order given, are 89%, 85%, 95%, 63%, 92%, 80%. **19. a.** $P(X = 8) = P(X = 6) = .3$ **b.** .7
21.

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



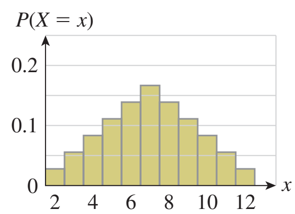
23.

x	0	1	4	9
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



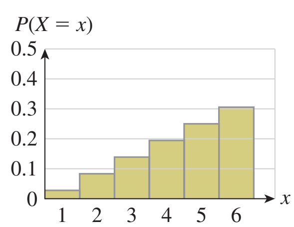
25.

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



27.

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$



29. a. 2000, 3000, 4000, 5000, 6000, 7000, 8000 (7000 is optional)
b.

x	2000	3000	4000	5000	6000	7000	8000
Freq.	2	1	1	1	2	0	3
$P(X = x)$	.2	.1	.1	.1	.2	.0	.3

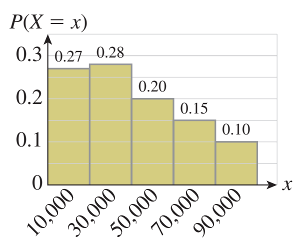
c. $P(X \le 5000) = .5$ **31.** The random variable is $X =$ mold count on a given day

X	750	2250	3750	5250
$P(X = x)$	11/16	2/16	1/16	2/16

33. a.

x	10,000	30,000	50,000	70,000	90,000
$P(X = x)$	.27	.28	.20	.15	.10

b. .25 Histogram:



35.

Class	1.1 − 2.0	2.1 − 3.0	3.1 − 4.0
Freq.	4	7	9

x	1.5	2.5	3.5
$P(X = x)$	.20	.35	.45

37. 95.5%

39.

x	3	2	1	0
$P(X = x)$	.0625	.6875	.125	.125

41. .75 The probability that a randomly selected small car is rated Good or Acceptable is .75. **43.** $P(Y \geq 2) = .50$, $P(Z \geq 2) \approx .53$, suggesting that medium SUVs are safer than small SUVs in frontal crashes **45.** Small cars **47.** .375

49.

x	1	2	3	4
$P(X = x)$	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

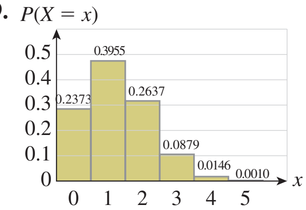
$P(X \geq 2) = 31/35 \approx .886$

51. Answers will vary. **53.** No; for instance, if X is the number of times you must toss a coin until heads comes up, then X is infinite but not continuous. **55.** By measuring the values of X for a large number of outcomes, and then using the estimated probability (relative frequency) **57.** Here is an example: let X be the number of days a diligent student waits before beginning to study for an exam scheduled in 10 days' time. **59.** Answers may vary. If we are interested in exact page-counts, then the number of possible values is very large and the values are (relatively speaking) close together, so using a continuous random variable might be advantageous. In general, the finer and more numerous the measurement classes, the more likely it becomes that a continuous random variable could be advantageous.

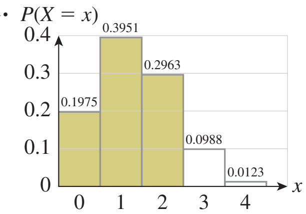
Section 8.2

1. .0729 **3.** .59049 **5.** .00001 **7.** .99144 **9.** .00856 **11.** .27648 **13.** .54432 **15.** .04096 **17.** .77414

19.



21.



$P(X \leq 2) = .8889$

23. .2637 **25.** .8926 **27.** .875 **29. a.** .0081 **b.** .08146 **31.** .41 **33. a.** .0519 **b.** Probability distribution (entries rounded to 4 decimal places):

x	0	1	2	3	4
$P(X = x)$	.6648	.2770	.0519	.0058	.0004
5	6	7	8	9	10
.0000	.0000	.0000	.0000	.0000	.0000

c. None **35.** .000298 **37.** .8321 **39. a.** 21 **b.** 20 **c.** The graph for $n = 50$ trials is more widely distributed than the graph for $n = 20$. **41.** 69 trials **43.** $.562 \times 10^{-5}$ **45.** .0266 Because there is only a 2.66% chance of detecting the disease in a given year, the government's claim seems dubious. **47.** No; in a sequence of Bernoulli trials, the occurrence of one success does not affect the probability of success on the next attempt. **49.** No; if life is a sequence of Bernoulli trials, then the occurrence of one misfortune ("success") does not affect the probability of a misfortune on the next trial. Hence, misfortunes may very well not "occur in threes." **51.** The probability of selecting a red marble changes after each selection, as the number of marbles left in the bag decreases. This violates the requirement that, in a sequence of Bernoulli trials, the probability of "success" does not change.

Section 8.3

1. $\bar{x} = 6$, median $= 5$, mode $= 5$
3. $\bar{x} = 3$, median $= 3.5$, mode $= -1$
5. $\bar{x} = -0.1875$, median $= 0.875$, every value is a mode
7. $\bar{x} = 0.2$, median $= -0.1$, mode $= -0.1$ **9.** Answers may vary. Two examples are: 0, 0, 0, 0, 0, 6 and 0, 0, 0, 1, 2, 3
11. 0.9 **13.** 21 **15.** −0.1 **17.** 3.5 **19.** 1 **21.** 4.472 **23.** 2.667 **25.** 2 **27.** 0.385 **29.** $\bar{x} = 5000$, $m = 5500$; 5500
31. $\bar{x} = \$426$, median $= \$425.50$, mode $= \$425$. Over the 10-business day period sampled, the price of gold averaged \$426 per ounce. It was above \$425.50 as many times as it was below that, and stood at \$425 per ounce for most of the days sampled.

33. a. 6.5; There were an average of 6.5 checkout lanes in a supermarket that was surveyed. **b.** $P(X < \mu) = .42$; $P(X > \mu) = .58$, and is thus larger. Most supermarkets have more than the average number of checkout lanes.

35.

X	5	10	15	20	25	35
***P*(*X*)**	.17	.33	.21	.19	.03	.07

$E(X) = 14.3$;

The average age of a school goer in 1998 was 14.3. **37.** 1687.5 **39.** \$41,000

41.

x	3	2	1	0
***P*(*X* = *x*)**	.0625	.6875	.125	.125

$E(X) = 1.6875$

y	3	2	1	0
***P*(*Y* = *y*)**	.1	.4	.4	.1

$E(Y) = 1.5$

Small cars

43. Large cars **45.** Expect to lose 5.3¢. **47.** 25.2 students **49. a.** 2 defective airbags **b.** 120 airbags

51.

x	1	2	3	4
***P*(*X* = *x*)**	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

$E(X) = 16/7 \approx 2.2857$ tents

53. FastForward: 3.97%; SolidState: 5.51%; SolidState gives the higher expected return. **55.** A loss of \$29,390 **57.** (A) **59.** He is wrong; for example, the collection 0, 0, 300 has mean 100 and median 0. **61.** No. The expected number of times you will hit the dart-board is the average number of times you will hit the bull's eye per 50 shots; the average of a set of whole numbers need not be a whole number. **63.** Wrong. It might be the case that only a small fraction of people in the class scored better than you but received exceptionally high scores that raised the class average. Suppose, for instance, that there are 10 people in the class. Four received 100%, you received 80%, and the rest received 70%. Then the class average is 83%, 5 people have lower scores than you, but only four have higher scores. **65.** No; the mean of a very large sample is only an *estimate* of the population mean. The means of larger and larger samples *approach* the population mean as the sample size increases. **67.** Wrong. The statement attributed to President Bush asserts that the mean tax refund would be \$1000, whereas the statements referred to as "The Truth" suggest that the *median* tax refund would be close to \$100 (and that the 31st percentile would be zero). **69.** Select a U.S. household at random, and let X be the income of that household. The expected value of X is then the population mean of all U.S. household incomes.

Section 8.4

1. $s^2 = 29$; $s = 5.39$ **3.** $s^2 = 12.4$; $s = 3.52$ **5.** $s^2 = 6.64$; $s = 2.58$ **7.** $s^2 = 13.01$; $s = 3.61$ **9.** 1.04 **11.** 9.43 **13.** 3.27 **15.** Expected value = 3.5, variance = 2.918, standard deviation = 1.71 **17.** Expected value = 1, variance = 0.5, standard deviation = 0.71 **19.** Expected value = 4.47, variance = 1.97, standard deviation = 1.40 **21.** Expected value = 2.67, variance = 0.36, standard deviation = 0.60 **23.** Expected value = 2, variance = 1.8, standard deviation = 1.34 **25. a.** $\bar{x} = 3$, $s = 3.54$ **b.** [0, 6.54] We must assume that the population distribution is bell-shaped and symmetric. **27. a.** $\bar{x} = 5.0$, $s = 0.6$ **b.** 3.8, 6.2 **29. a.** $\bar{x} = 5000$, $s \approx 2211$ **b.** 2789, 7211, 60% **31. a.** 2.18 **b.** [11.22, 24.28] **c.** 100%; Empirical rule **33.** $\mu = 1.5$, $\sigma = 1.43$; 100% **35.** $\mu = 40.6$, $\sigma \approx 26$; \$52,000 **37. a.** $\mu \approx 30.2$ yrs. old, $\sigma = 11.78$ years **b.** 18–42 **39.** At most 6.25% **41.** At most; 12.5% **43. a.** $\mu = 25.2$, $\sigma = 3.05$ **b.** 31 **45. a.** $\mu = 780$, $\sigma \approx 13.1$ **b.** 754, 806 **47. a.** $\mu = 6.5$, $\sigma^2 = 4.0$, $\sigma = 2.0$ **b.** [2.5, 10.5]; 3 checkout lanes **49.** \$10,700 or less **51.** \$65,300 or more **53.** U.S. **55.** U.S. **57.** 16% **59.** 0–\$76,000 **61.** $\mu = 12.56\%$, $\sigma \approx 1.8885\%$ **63.** 78%; The empirical rule predicts 68%. The associated probability distribution is roughly bell-shaped but not symmetric. **65.** 96%. Chebyshev's rule is valid, since it predicts that *at least* 75% of the scores are in this range. **67.** (B), (D) **69.** The sample standard deviation is bigger; the formula for sample standard deviation involves division by the smaller term $n - 1$ instead of n, which makes the resulting number larger. **71.** The grades in the first class were clustered fairly close to 75. By Chebyshev's inequality, at least 88% of the class had grades in the range 60–90. On the other hand, the grades in the second class were widely dispersed. The second class had a much wider spread of ability than did the first class. **73.** The variable must take on only the value 10, with probability 1. **75.** $(y - x)/2$

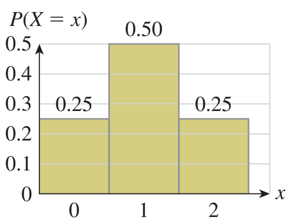
Section 8.5

1. .1915 **3.** .5222 **5.** .6710 **7.** .2417 **9.** .8664 **11.** .8621 **13.** .2286 **15.** .3830 **17.** .5028 **19.** .35 **21.** .05 **23.** .3830 **25.** .6687 **27.** 26% **29.** 29,600,000 **31.** 0 **33.** About 6680 **35.** 28% **37.** 5% **39.** The U.S. **41.** Wechsler. Because this test has a smaller standard deviation, a greater percentage of scores fall within 20 points of the mean. **43.** This is surprising, because the time between failures was more than 5 standard deviations away from the mean, which happens with an extremely small probability. **45.** .6103 **47.** $.6103 \times .5832 \approx .3559$ **49.** .6255 **51.** .7257 **53.** .8708 **55.** .0029 **57.** Probability that a person will say Goode = .54. Probability that Goode polls more than 52% ≈ .8925. **59.** 23.4 **61.** When the distribution is normal **63.** Neither. They are equal. **65.** $1/(b - a)$ **67.** A normal distribution with standard deviation 0.5, because it is narrower near the mean, but must enclose the same amount of area as the standard curve, and so it must be higher.

Chapter 8 Review

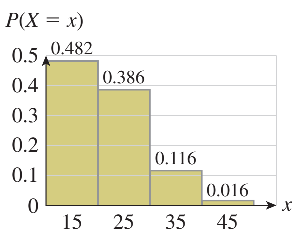
1.

x	0	1	2
$P(X = x)$	1/4	1/2	1/4



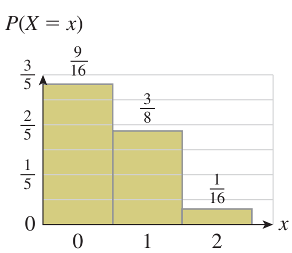
3.

x	15	25	35	45
$P(X = x)$	.482	.386	.116	.016



5.

x	0	1	2
$P(X = x)$	9/16	6/16	1/16



7. Two examples are: 0, 0, 0, 4 and −1, −1, 1, 5 **9.** An example is −1, −1, −1, 1, 1, 1 **11.** .4165 **13.** .3232 **15.** .7330

17.

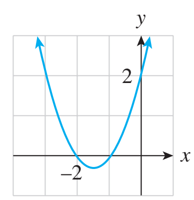
x	−3	−2	−1	0	1	2	3
$P(X = x)$	1/16	2/16	3/16	4/16	3/16	2/16	1/16

$\mu = 0$, $\sigma = 1.5811$; within 1.3 standard deviations of the mean. **19.** [40, 160] **21.** .0668 **23.** .7888 **25.** .0000 **27. a.** $27,210 **b.** False; let X = price and Y = weekly sales. Then weekly Revenue = XY. However, $27{,}210 \neq 12.15 \times 2{,}620$. In other words, $E(XY) \neq E(X)E(Y)$. **29.** Between 2.431 and 7.569 orders per million residents. The empirical rule does not apply because the distribution is not symmetric. **31.** .190 **33.** .060 **35.** 2.5 **37.** .873 **39.** Using normal distribution table: 364,000 people. More accurate answer: 378,000 people **41.** 148

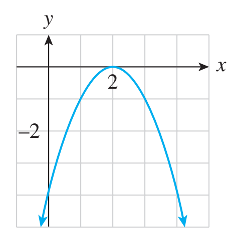
Chapter 9

Section 9.1

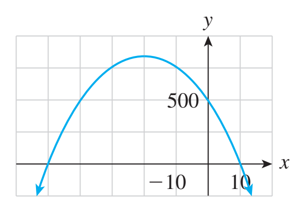
1. Vertex: (−3/2, −1/4); y-intercept: 2; x-intercepts: −2, −1



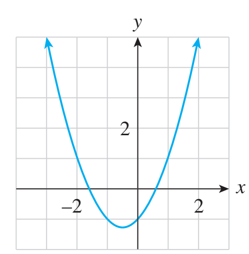
3. Vertex: (2,0); y-intercept: −4; x-intercept: 2



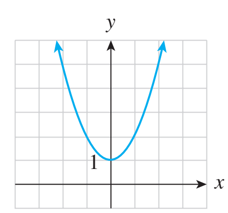
5. Vertex: (−20, 900); y-intercept: 500; x-intercepts: −50, 10



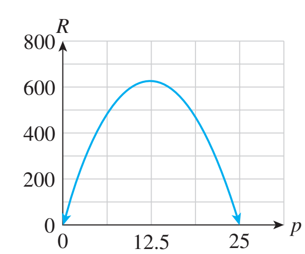
7. Vertex: (−1/2, −5/4); y-intercept: −1; x-intercepts: $-1/2 \pm \sqrt{5}/2$



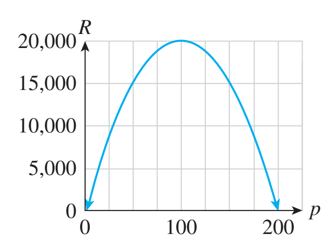
9. Vertex: (0, 1); y-intercept: 1; No x-intercepts



11. $R = -4p^2 + 100p$; Maximum revenue when $p = \$12.50$

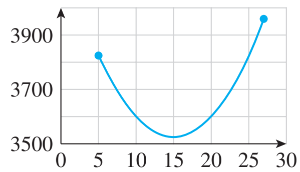


13. $R = -2p^2 + 400p$; Maximum revenue when $p = \$100$

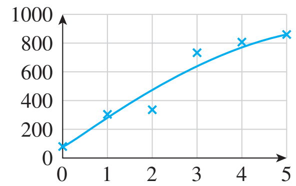


15. $y = -0.7955x^2 + 4.4591x - 1.6000$
17. $y = -1.1667x^2 - 6.1667x - 3.0000$
19. a. Positive because the data suggest a curve that is concave up. **b.** (C) **c.** 1995. The parabola rises to the left of the vertex and thus predicts increasing trade as we go back in time, contradicting history.

21. 1985 ($t = 15$); 3525 pounds



23. 5000 pounds. The model is not trustworthy for vehicle weights larger than 5000 pounds, because it predicts increasing fuel economy with increasing weight, and 5000 is close to the upper limit of the domain of the function.
25. Maximum revenue when $p = \$140$, $R = \$9800$
27. Maximum revenue with 70 houses, $R = \$9{,}800{,}000$
29. a. $q = -560x + 1400$; $R = -560x^2 + 1400x$
b. $P = -560x^2 + 1400x - 30$; $x = \$1.25$; $P = \$845$ per month
31. $C = -200x + 620$; $P = -400x^2 + 1400x - 620$
$x = \$1.75$ per log-on; $P = \$605$ per month
33. a. $q = -10p + 400$ **b.** $R = -10p^2 + 400p$
c. $C = -30p + 4200$
d. $P = -10p^2 + 430p - 4200$; $p = \$21.50$
35. $C(t) = 2.7t^2 - 4.5t + 50$; \$120.2 billion, which agrees with the actual value to the nearest \$1 billion.
37. a. $S(t) = -12.27t^2 + 227.23t + 64.39$



b. 986,000 units **c.** Mathematical regression cannot reliably be used to make predictions about sales. (Answers will vary.)
39. The x-coordinate of the vertex represents the unit price that leads to the maximum revenue, the y-coordinate of the vertex gives the maximum possible revenue, the x-intercepts give the unit prices that result in zero revenue, and the y-intercept gives the revenue resulting from zero unit price (which is obviously zero).
41. Graph the data to see whether the points suggest a curve rather than a straight line. If the curve suggested by the graph is concave up or concave down, then a quadratic model would be a likely candidate. **43.** If $q = mp + b$ (with $m < 0$), then the revenue is given by $R = pq = mp^2 + bp$. This is the equation of a parabola with $a = m < 0$, and so is concave down. Thus the vertex is the highest point on the parabola, showing that there is a single highest value for R, namely, the y-coordinate of the vertex.
45. Because $R = pq$, the demand must be given by

$$q = \frac{R}{p} = \frac{-50p^2 + 60p}{p} = -50p + 60.$$

Section 9.2

1. `4^x`

x	−3	−2	−1	0	1	2	3
$f(x)$	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

3. `3^(-x)`

x	−3	−2	−1	0	1	2	3
$f(x)$	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

5. `2*2^x` or `2*(2^x)`

x	−3	−2	−1	0	1	2	3
$g(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

7. `-3*2^(-x)`

x	−3	−2	−1	0	1	2	3
$h(x)$	−24	−12	−6	−3	$-\frac{3}{2}$	$-\frac{3}{4}$	$-\frac{3}{8}$

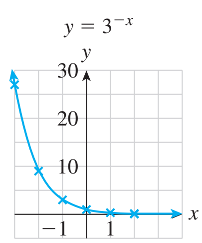
9. `2^x-1`

x	−3	−2	−1	0	1	2	3
$r(x)$	$-\frac{7}{8}$	$-\frac{3}{4}$	$-\frac{1}{2}$	0	1	3	7

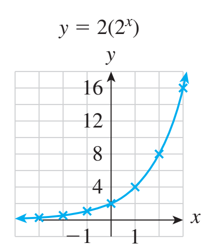
11. `2^(x-1)`

x	−3	−2	−1	0	1	2	3
$s(x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

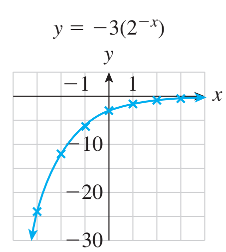
13.



15.



17.



19. Both; $f(x) = 4.5(3^x)$, $g(x) = 2(1/2)^x$, or $2(2^{-x})$
21. Neither **23.** g; $g(x) = 4(0.2)^x$
25. `e^(-2*x)` or `EXP(-2*x)`

x	-3	-2	-1	0	1	2	3
$f(x)$	403.4	54.60	7.389	1	0.1353	0.01832	0.002479

27. `1.01*2.02^(-4*x)`

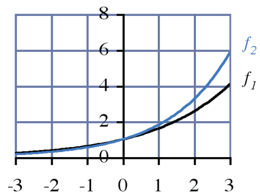
x	-3	-2	-1	0	1	2	3
$h(x)$	4662	280.0	16.82	1.01	0.06066	0.003643	0.0002188

29. `50*(1+1/3.2)^(2*x)`

x	-3	-2	-1	0	1	2	3
$r(x)$	9.781	16.85	29.02	50	86.13	148.4	255.6

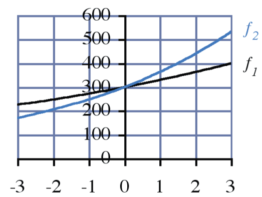
31. `2^(x-1)`; **not** `2^x-1` **33.** `2/(1-2^(-4*x))`; **not** `2/1-2^-4*x`; **not** `2/1-2^(-4*x)`
35. `(3+x)^(3*x)/(x+1)` or `((3+x)^(3*x))/(x+1)`; **not** `(3+x)^(3*x)/x+1`; **not** `(3+x^(3*x))/(x+1)`
37. `2*e^((1+x)/x)` or `2*EXP((1+x)/x)`; **not** `2*e^1+x/x`; **not** `2*e^(1+x)/x`; **not** `2*EXP(1+x)/x`

39.



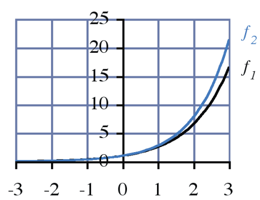
y1 = 1.6^x y2 = 1.8^x

41.



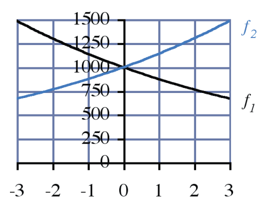
y1 = 300*1.1^x
y2 = 300*1.1^(2*x)

43.



Y1 = 2.5^(1.02*x)
y2 = e^(1.02*x) or
exp(1.02*x)

45.



y1 = 1000*1.045^(-3*x)
y2 = 1000*1.045^(3*x)

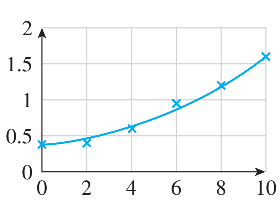
47. $f(x) = 500(0.5)^x$ **49.** $f(x) = 10(3)^x$ **51.** $f(x) = 500(0.45)^x$ **53.** $f(x) = -100(1.1)^x$ **55.** $y = 4(3^x)$ **57.** $y = -1(0.2^x)$ **59.** $y = 2.1213(1.4142^x)$ **61.** $y = 3.6742(0.9036^x)$ **63.** $f(t) = 5000e^{0.10t}$ **65.** $f(t) = 1000e^{-0.063t}$ **67.** $y = 1.0442(1.7564)^x$ **69.** $y = 15.1735(1.4822)^x$ **71.** $y = 1000(2^{t/3})$; 65,536,000 bacteria after 2 days **73.** $A(t) = 5000(1.0439)^t$; £6198 **75.** At the beginning of 2014 **77.** 31.0 grams, 9.25 grams, 2.76 grams **79.** 20,000 years **81.** 53 mg **83. a.** $P = 40t + 360$ **b.** $P = 360(1.1006)^t$. Neither model is applicable. **85. a.** $P = 180(1.01121)^t$ million **b.** 4 decimal places **c.** 351 million **87. a.** $y = 50{,}000(1.5^{t/2})$, t = time in years since two years ago **b.** 91,856 tags **89.** $491.82

91. a.

Year	1950	2000	2050	2100
$C(t)$ parts per million	561	669	799	953

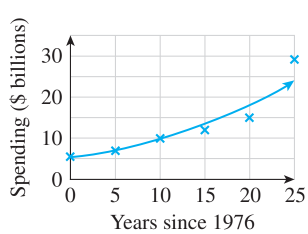
b. 2010 ($t = 260$)

93. a. $P(t) = 0.339(1.169)^t$. Graph:



b. $1.9 million

95. a. $y = 5.4433(1.0609)^t$. Graph:



b. 609% **c.** $16 billion

97. (B) **99.** Exponential functions of the form $f(x) = A(b^x)\,(b > 0)$ increase rapidly for large values of x. In real-life situations, such as population growth, this model is reliable only for relatively short periods of growth. Eventually, population growth tapers off because of pressures such as limited resources and overcrowding. **101.** Linear functions better: cost models where there is a fixed cost and a variable cost; simple interest, where interest is paid on the original amount invested. Exponential models better: compound interest, population growth. (In both of these, the rate of growth depends on the present number of items, rather than on some fixed quantity.) **103.** Take the ratios y_2/y_1 and y_3/y_2. If they are the same, the points fit on an exponential curve. **105.** This reasoning is suspect—the bank need not use its computer resources to update all the accounts every minute, but can instead use the continuous compounding formula to calculate the balance in any account at any time.

Section 9.3

1.

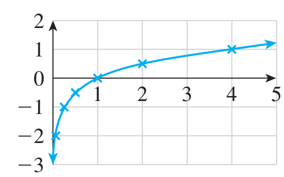
Logarithmic Form	$\log_{10} 10{,}000 = 4$	$\log_4 16 = 2$	$\log_3 27 = 3$	$\log_5 5 = 1$	$\log_7 1 = 0$	$\log_4 \frac{1}{16} = -2$

3.

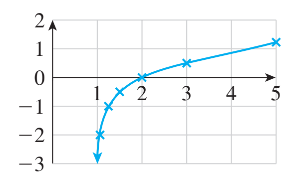
Exponential Form	$(0.5)^2 = 0.25$	$5^0 = 1$	$10^{-1} = 0.1$	$4^3 = 64$	$2^8 = 256$	$2^{-2} = \frac{1}{4}$

5. 1.4650 **7.** −1.1460 **9.** −0.7324 **11.** 6.2657

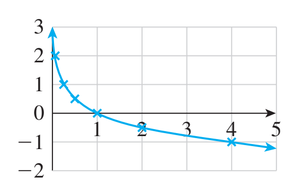
13.



15.



17.

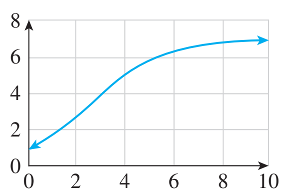


19. $Q = 1000e^{-t \ln 2}$ **21.** $Q = 1000e^{t(\ln 2)/2}$ **23.** Doubling time = $2 \ln 2$ **25.** Half-life = $(\ln 2)/4$ **27.** $f(x) = 4(7.389)^x$ **29.** $f(t) = 2.1e^{0.0009995t}$ **31.** $f(t) = 10e^{-0.01309}$ **33.** 3.36 years **35.** 11 years **37.** 23.1% **39.** 63,000 years old **41.** 8 years **43.** 151 months **45.** 12 years **47.** 13.08 years **49.** 1600 years **51. a.** $b = 3^{1/6} \approx 1.20$ **b.** 3.8 months **53. a.** $Q(t) = Q_0 e^{-0.139t}$ **b.** 3 years **55.** 2360 million years **57.** 3.2 hours **59.** 3.89 days **61. a.** $P(t) = 6.591 \ln t - 17.69$ **b.** 1 digit **c.** (A) **63.** $M(t) = 11.622 \ln t - 7.1358$. The model is unsuitable for large values of t since, for sufficiently large values of t, $M(t)$ will eventually become larger than 100%. **65. a.** About 1.259×10^{24} ergs **b.** about 2.24% **d.** 1000 **67. a.** 75 dB, 69 dB, 61 dB **b.** $D = 95 - 20 \log r$ **c.** 57,000 feet **69.** The logarithm of a negative number, were it defined, would be the power to which a base must be raised to give that negative number. But raising a

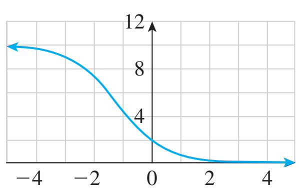
base to a power never results in a negative number, so there can be no such real number as the logarithm of a negative number.
71. $\log_4 y$ **73.** 8 **75.** x **77.** Any logarithmic curve $y = \log_b t + C$ will eventually surpass 100%, and hence not be suitable as a long-term predictor of market share. **79.** Time is increasing logarithmically with population; Solving $P = Ab^t$ for t gives $t = \log_b(P/A) = \log_b P - \log_b A$, which is of the form $t = \log_b P + C$.

Section 9.4

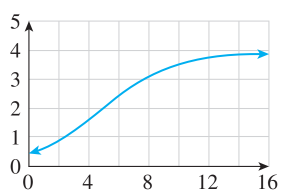
1. $N = 7, A = 6, b = 2$; `7/(1+6*2^-x)`



3. $N = 10, A = 4, b = 0.3$; `10/(1+4*0.3^-x)`



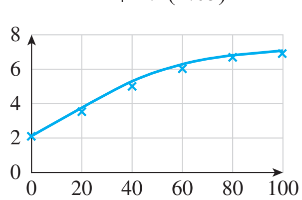
5. $N = 4, A = 7, b = 1.5$; `4/(1+7*1.5^-x)`

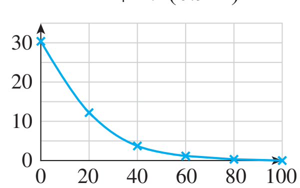


7. $f(x) = \dfrac{200}{1 + 19(2^{-x})}$ **9.** $f(x) = \dfrac{6}{1 + 2^{-x}}$

11. (B) **13.** (B) **15.** (C)

17. $y = \dfrac{7.2}{1 + 2.4(1.05)^{-x}}$ **19.** $y = \dfrac{97}{1 + 2.2(0.942)^{-x}}$



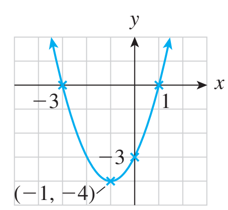


21. a. (A) **b.** 20% per year **23. a.** 91% **b.** $P(x) \approx 14.33(1.05)^x$ **c.** \$38,000 **25.** $N(t) = \dfrac{10{,}000}{1 + 9(1.25)^{-t}}$; $N(7) \approx 3463$ cases **27.** $N(t) = \dfrac{3000}{1 + 29(2^{1/5})^{-t}}$; $t = 16$ days **29. a.** $A(t) = \dfrac{6.3}{1 + 4.8(1.2)^{-t}}$; 6300 articles **b.** 5200 articles **31. a.** $N(t) = \dfrac{82.8}{1 + 21.8(7.14)^{-t}}$. The model predicts that book sales will level off at around 82.8 million books per year. **b.** Not consistent; 15% of the market is represented by more than double the predicted value. This shows the difficulty in making long-term predictions from regression models obtained from a small amount of data. **c.** 2001

33. $N(t) = \dfrac{5}{1 + 1.080(1.056)^{-t}}$; $t = 17$, or 2010. **35.** Just as diseases are communicated via the spread of a pathogen (such as a virus), new technology is communicated via the spread of information (such as advertising and publicity). Further, just as the spread of a disease is ultimately limited by the number of susceptible individuals, so the spread of a new technology is ultimately limited by the size of the potential market. **37.** It can be used to predict where the sales of a new commodity might level off.

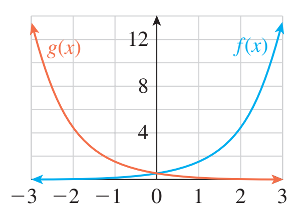
Chapter 9 Review

1.

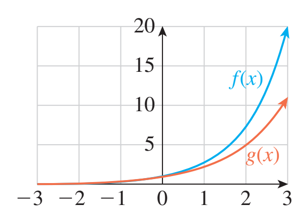


3. f: $f(x) = 5(1/2)^x$, or $5(2^{-x})$

5.



7.

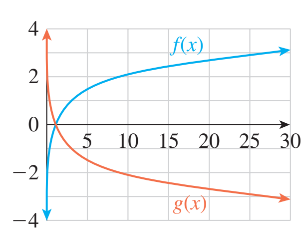


9. \$3484.85 **11.** \$3705.48 **13.** \$3485.50

15. $f(x) = 4.5(9^x)$ **17.** $f(x) = \dfrac{2}{3}3^x$ **19.** $-\dfrac{1}{2}\log_3 4$

21. $\dfrac{1}{3}\log 1.05$

23.



25. $Q = 5e^{-0.00693t}$ **27.** $Q = 2.5e^{0.347t}$ **29.** 10.2 years

31. 10.8 years **33.** $f(x) = \dfrac{900}{1 + 8(1.5)^{-x}}$

35. $f(x) = \dfrac{20}{1 + 3(0.8)^{-x}}$ **37. a.** \$8500 per month; an average of approximately 2100 hits per day **b.** \$29,049 per month **c.** The fact that −0.000005, the coefficient of c^2, is negative. **39.** $R = -60p^2 + 950p$; $p = \$7.92$ per novel, Monthly revenue = \$3760.42 **41. a.** 10, 34 **b.** About 360,000 pounds **43.** 2008 **45.** 32.8 hours **47.** (C)

Chapter 10

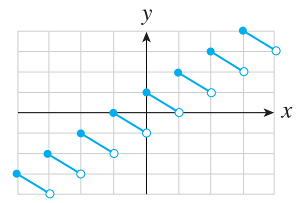
Section 10.1

1. 0 **3.** 4 **5.** Does not exist **7.** 1.5 **9.** 0.5 **11.** Diverges to $+\infty$ **13.** 0 **15.** 1 **17.** 0 **19. a.** -2 **b.** -1 **21. a.** 2 **b.** 1 **c.** 0 **d.** $+\infty$ **23. a.** 0 **b.** 2 **c.** -1 **d.** Does not exist **e.** 2 **f.** $+\infty$ **25. a.** 1 **b.** 1 **c.** 2 **d.** Does not exist **e.** 1 **f.** 2 **27. a.** 1 **b.** $+\infty$ **c.** $+\infty$ **d.** $+\infty$ **e.** not defined **f.** -1 **29. a.** -1 **b.** $+\infty$ **c.** $-\infty$ **d.** Does not exist **e.** 2 **f.** 1
31. 7.0; In the long term, the number of research articles in *Physics Review* written by researchers in Europe approaches 7000 per year.
33. 470. This suggests that students whose parents earn an exceptionally large income score an average of 470 on the SAT verbal test. **35.** $\lim_{t\to 1^-} C(t) = 0.06$, $\lim_{t\to 1^+} C(t) = 0.08$, so $\lim_{t\to 1} C(t)$ does not exist. **37.** $\lim_{t\to +\infty} I(t) = +\infty$, $\lim_{t\to +\infty}(I(t)/E(t)) \approx 2.5$. In the long term, U.S. imports from China will rise without bound and be 2.5 times U.S. exports to China. In the real world, imports and exports cannot rise without bound. Thus, the given models should not be extrapolated far into the future. **39.** $\lim_{t\to +\infty} n(t) \approx 80$. Online book sales can be expected to level off at 80 million per year in the long term. **41.** To approximate $\lim_{x\to a} f(x)$ numerically, choose values of x closer and closer to, and on either side of $x = a$, and evaluate $f(x)$ for each of them. The limit (if it exists) is then the number that these values of $f(x)$ approach. A disadvantage of this method is that it may never give the exact value of the limit, but only an approximation. (However, we can make this as accurate as we like.) **43.** It is possible for $\lim_{x\to a} f(x)$ to exist even though $f(a)$ is not defined. An example is $\lim_{x\to 1} \dfrac{x^2 - 3x + 2}{x - 1}$.
45. Any situation in which there is a sudden change can be modeled by a function in which $\lim_{t\to a^+} f(t)$ is not the same as $\lim_{t\to a^-} f(t)$ One example is the value of a stock market index before and after a crash: $\lim_{t\to a^-} f(t)$ is the value immediately before the crash at time $t = a$, while $\lim_{t\to a^+} f(t)$ is the value immediately after the crash. Another example might be the price of a commodity that has suddenly increased from one level to another.
47. An example is $f(x) = (x - 1)(x - 2)$.

Section 10.2

1. Continuous on its domain **3.** Continuous on its domain **5.** Discontinuous at $x = 0$ **7.** Discontinuous at $x = -1$ **9.** Continuous on its domain **11.** Discontinuous at $x = -1$ and 0 **13.** (A), (B), (D), (E) **15.** 0 **17.** -1 **19.** No value possible **21.** -1 **23.** Continuous on its domain **25.** Continuous on its domain **27.** Discontinuity at $x = 0$ **29.** Discontinuity at $x = 0$ **31.** Continuous on its domain **33.** Not unless the domain of the function consists of all real numbers. (It is impossible for a function to be continuous at points not in its domain.) For example, $f(x) = 1/x$ is continuous on its domain—the set of nonzero real numbers—but not at $x = 0$. **35.** True. If the graph of a function has a break in its graph at any point a, then it cannot be continuous at the point a.

37. Answers may vary.



39. Answers may vary. The price of OHaganBooks.com stock suddenly drops by \$10 as news spreads of a government investigation. Let $f(x) =$ Price of OHaganBooks.com stock.

Section 10.3

1. $x = 1$ **3.** 2 **5.** 1 **7.** 2 **9.** 0 **11.** 6 **13.** 4 **15.** 2 **17.** 0 **19.** 0 **21.** 12 **23.** Diverges to $+\infty$ **25.** Does not exist; left and right (infinite) limits differ **27.** 3/2 **29.** 1/2 **31.** Diverges to $+\infty$ **33.** 0 **35.** 3/2 **37.** 1/2 **39.** Diverges to $-\infty$ **41.** 0 **43.** Discontinuity at $x = 0$ **45.** Continuous everywhere **47.** Discontinuity at $x = 0$ **49.** Discontinuity at $x = 0$ **51. a.** 0.49, 1.16. Shortly before 1999, annual advertising expenditures were close to \$0.49 billion. Shortly after 1999, annual advertising expenditures were close to \$1.16 billion. **b.** Not continuous; Movie advertising expenditures jumped suddenly in 1999. **53.** 1.59; If the trend continues indefinitely, the annual spending on police will be 1.59 times the annual spending on courts in the long run. **55.** $\lim_{t\to +\infty} I(t) = +\infty$, $\lim_{t\to +\infty}(I(t)/E(t)) = 2.5$. In the long term, U.S. imports from China will rise without bound and be 2.5 times U.S. exports to China. In the real world, imports and exports cannot rise without bound. Thus, the given models should not be extrapolated far into the future. **57.** $\lim_{t\to +\infty} p(t) = 100$. The percentage of children who learn to speak approaches 100% as their age increases. **59.** Yes; $\lim_{t\to 8^-} C(t) = \lim_{t\to 8^+} C(t) = 1.24$. **61.** To evaluate $\lim_{x\to a} f(x)$ algebraically, first check whether $f(x)$ is a closed-form function. Then check whether $x = a$ is in its domain. If so, the limit is just $f(a)$; that is, it is obtained by substituting $x = a$. If not, then try to first simplify $f(x)$ in such a way as to transform it into a new function such that $x = a$ is in its domain, and then substitute. A disadvantage of this method is that it is sometimes extremely difficult to evaluate limits algebraically, and rather sophisticated methods are often needed. **63.** She is wrong. Closed-form functions are continuous only at points in their domains, and $x = 2$ is not in the domain of the closed-form function $f(x) = 1/(x - 2)^2$. **65.** The statement may not be true, for instance, if $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x - 1 & \text{if } x \ge 0 \end{cases}$, then $f(0)$ is defined and equals -1, and yet $\lim_{x\to 0} f(x)$ does not exist. The statement can be corrected by requiring that f be a closed-form function: "If f is a closed form function, and $f(a)$ is defined, then $\lim_{x\to a} f(x)$ exists and equals $f(a)$." **67.** Answers may vary, for example

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is any number other than 1 or 2} \\ 1 & \text{if } x = 1 \text{ or } 2 \end{cases}$$

Section 10.4

1. −3 **3.** 0.3 **5.** −$25,000 per month **7.** −200 items per dollar **9.** $1.33 per month **11.** 0.75 percentage point increase in unemployment per 1 percentage point increase in the deficit **13.** 4 **15.** 2 **17.** 7/3

19.

h	Ave. Rate of Change
1	2
0.1	0.2
0.01	0.02
0.001	0.002
0.0001	0.0002

21.

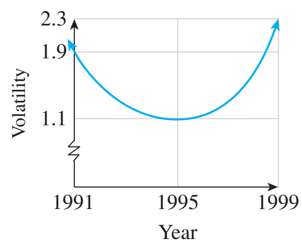
h	Ave. Rate of Change
1	−0.1667
0.1	−0.2381
0.01	−0.2488
0.001	−0.2499
0.0001	−0.24999

23.

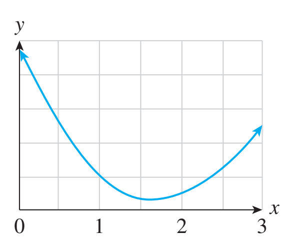
h	Ave. Rate of Change
1	9
0.1	8.1
0.01	8.01
0.001	8.001
0.0001	8.0001

25. a. −0.25 million people per year. During the period 2000–2004, employment in the U.S. decreased at an average rate of 0.25 million people per year. **b.** Zero people per year. During the period 1999–2002 the average rate of change of employment in the U.S. was zero people per year. **27. a.** 1998–2000. The number of companies that invested in venture capital each year was increasing most rapidly during the period 1998–2000, when it grew at an average rate of 650 companies per year. **b.** 1999–2001. The number of companies that invested in venture capital each year was decreasing most rapidly during the period 1999–2001, when it decreased at an average rate of 50 companies per year.
29. a. [3, 5]; −0.25 thousand articles per year. During the period 1993–1995, the number of articles authored by U.S. researchers decreased at an average rate of 250 articles per year. **b.** Percentage rate ≈ −0.1765, Average rate = −0.09 thousand articles/year. Over the period 1993–2003, the number of articles authored by U.S. researchers decreased at an average rate of 90 per year, representing a 17.65% decrease over that period. **31. a.** 75 teams per year **b.** Decreased **33. a.** 250 million transactions per year, –150 million transactions per year, 50 million transactions per year. Over the period January 2000–January 2001, the (annual) number of online shopping transactions in the U.S. increased at an average rate of 250 million per year. From January 2001 to January 2002, this number decreased at an average rate of 150 million per year. From January 2000 to January 2002, this number increased at an average rate of 50 million per year. **b.** The average rate of change of $N(t)$ over [0, 2] is the average of the rates of change over [0, 1] and [1, 2]. **35. a.** (C) **b.** (A) **c.** (B)
d. Approximately −0.0063 (to two significant digits) billion dollars per year, (−$6,300,000 per year). This is much less than the (positive) slope of the regression line, 0.0125 ≈ 0.013 billion dollars per year, ($13,000,000 per year).

37. Answers may vary



39. The index was increasing at an average rate of 300 points per day. **41.** $0.08 per year. The value of the euro in U.S. dollars was growing at an average rate of about $0.08 per year over the period June 2000–June 2004. **43. a.** 8.85 manatee deaths per 100,000 boats; 23.05 manatee deaths per 100,000 boats **b.** More boats result in more manatee deaths per additional boat. **45. a.** $305 million per year; Over the period 1997–1999, annual advertising revenues increased at an average rate of $305 million per year. **b.** (A) **c.** $590 million per year; The model projects annual advertising revenues to increase by $590 million per year in 2000. **47. a.** −0.88, −0.79, −0.69, −0.60, −0.51, −0.42 **b.** For household incomes between $40,000 and $40,500, the poverty rate decreases at an average rate of 0.69 percentage points per $1000 increase in the median household income. **c.** (B) **d.** (B). **49.** The average rate of change of f over an interval $[a, b]$ can be determined numerically, using a table of values; graphically, by measuring the slope of the corresponding line segment through two points on the graph; or algebraically, using an algebraic formula for the function. Of these, the least precise is the graphical method, because it relies on reading coordinates of points on a graph. **51.** Answers will vary.



53. 6 units of quantity A per unit of quantity C **55.** (A)
57. Yes. Here is an example:

Year	2000	2001	2002	2003
Revenue ($ billion)	10	20	30	5

59. (A)

Section 10.5

1. 6 **3.** −5.5
5.

h	1	0.1	0.01
Ave. rate	39	39.9	39.99

Instantaneous Rate = 40 rupees per day

7.

h	1	0.1	0.01
Ave. rate	140	66.2	60.602

Instantaneous Rate = 60 rupees per day

9.

h	10	1
C_{ave}	4.799	4.7999

$C'(1{,}000) = \$4.8$ per item

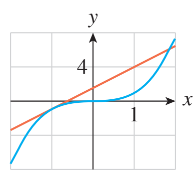
11.

h	10	1
C_{ave}	99.91	99.90

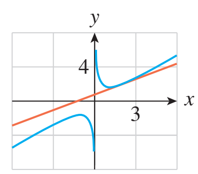
$C'(100) = \$99.90$ per item

13. a. R **b.** P **15. a.** P **b.** R **17. a.** Q **b.** P **19.** 1/2 **21.** 0 **23. a.** Q **b.** R **c.** P **25. a.** R **b.** Q **c.** P **27. a.** (1, 0) **b.** None **c.** (−2, 1) **29. a.** (−2, 0.3), (0, 0), (2, −0.3) **b.** None **c.** None **31.** $(a, f(a))$; $f'(a)$ **33.** (B) **35. a.** (A) **b.** (C) **c.** (B) **d.** (B) **e.** (C) **37.** −2 **39.** −1.5 **41.** −5 **43.** 16 **45.** 0 **47.** −0.0025

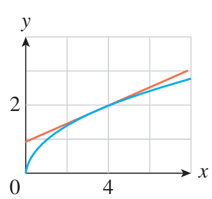
49. a. 3 **b.** $y = 3x + 2$



51. a. $\frac{3}{4}$ **b.** $y = \frac{3}{4}x + 1$

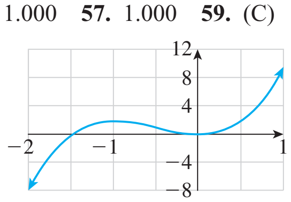


53. a. $\frac{1}{4}$ **b.** $y = \frac{1}{4}x + 1$



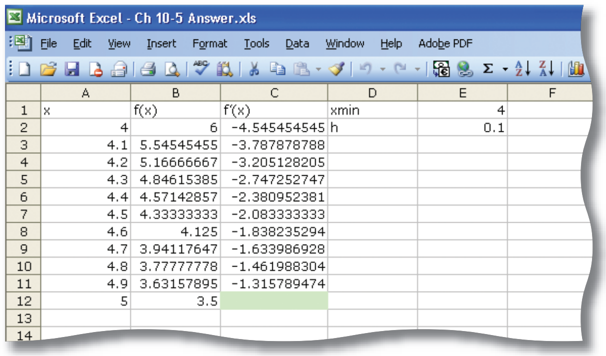
55. 1.000 **57.** 1.000 **59.** (C) **61.** (A) **63.** (F)

65.



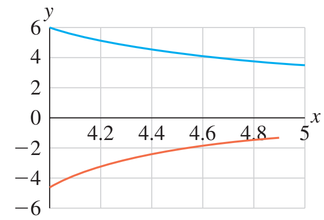
$x = -1.5, x = 0$

67. Note: Answers depend on the form of technology used. Excel ($h = 0.1$):



	A	B	C	D	E	F
1	x	f(x)	f'(x)	xmin	4	
2	4	6	-4.545454545	h	0.1	
3	4.1	5.54545455	-3.787878788			
4	4.2	5.16666667	-3.205128205			
5	4.3	4.84615385	-2.747252747			
6	4.4	4.57142857	-2.380952381			
7	4.5	4.33333333	-2.083333333			
8	4.6	4.125	-1.838235294			
9	4.7	3.94117647	-1.633986928			
10	4.8	3.77777778	-1.461988304			
11	4.9	3.63157895	-1.315789474			
12	5	3.5				
13						
14						

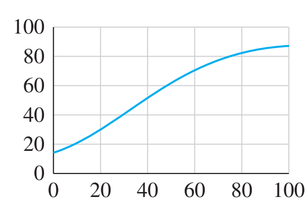
Graphs:



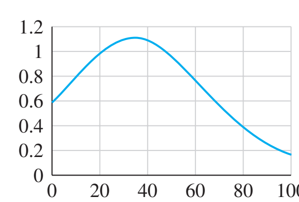
The top curve is $y = f(x)$; the bottom curve is $y = f'(x)$.

69. $q(100) = 50{,}000$, $q'(100) = -500$. A total of 50,000 pairs of sneakers can be sold at a price of \$100, but the demand is decreasing at a rate of 500 pairs per \$1 increase in the price. **71. a.** Sales in 2000 were approximately 160,000 pools per year, and increasing at a rate of 6000 per year. **b.** Decreasing because the slope is decreasing. **73. a.** (B) **b.** (B) **c.** (A) **d.** 1992 **e.** 0.05. In 1996, the total number of state prisoners was increasing at a rate of approximately 50,000 prisoners per year. **75. a.** −96 ft/sec **b.** −128 ft/sec **77. a.** \$0.044 per year. The value of the euro was increasing at an average rate of about \$0.044 per year over the period January 2000–January 2004. **b.** −\$0.10 per year. In January, 2000, the value of the euro was decreasing at a rate of about \$0.10 per year. **c.** The value of the euro was decreasing in January 2000, and then began to increase. **79. a.** \$305 million per year **b.** (A) **c.** \$685 million/year. In December 2000, AOL's advertising revenue was projected to be increasing at a rate of \$685 million per year. **81.** $A(0) = 4.5$ million; $A'(0) = 60{,}000$ **83. a.** 60% of children can speak at the age of 10 months. At the age of 10 months, this percentage is increasing by 18.2 percentage points per month. **b.** As t increases, p approaches 100 percentage points (all children eventually learn to speak), and dp/dt approaches zero because the percentage stops increasing. **85.** $S(5) \approx 109$, $\left.\frac{dS}{dt}\right|_{t=5} \approx 9.1$. After 5 weeks, sales are 109 pairs of sneakers per week, and sales are increasing at a rate of 9.1 pairs per week each week. **87. a.** $P(50) \approx 62$, $P'(50) \approx 0.96$; 62% of U.S. households with an income of \$50,000 have a computer. This percentage is increasing at a rate of 0.96 percentage points per \$1000 increase in household income. **b.** P' decreases toward zero.

Graphs:

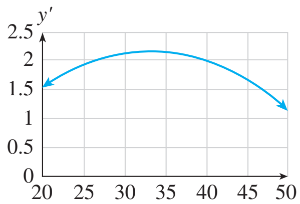


Graph of P



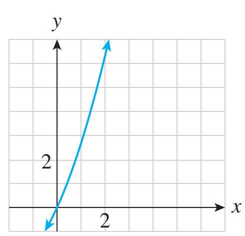
Graph of P'

89. a. (D) **b.** 33 days after the egg was laid **c.** 50 days after the egg was laid. Graph:



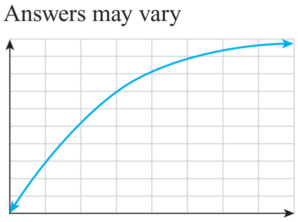
91. $L(0.95) = 31.2$ meters and $L'(0.95) = -304.2$ meters/warp. Thus, at a speed of warp 0.95, the spaceship has an observed length of 31.2 meters and its length is decreasing at a rate of 304.2 meters per unit warp, or 3.042 meters per increase in speed of 0.01 warp. **93.** The difference quotient is not defined when $h = 0$ because there is no such number as $0/0$. **95.** The derivative is positive and decreasing toward zero. **97.** Company B. Although the company is currently losing money, the derivative is positive, showing that the profit is increasing. Company A, on the other hand, has profits that are declining. **99.** (C) is the only graph in which the instantaneous rate of change on January 1 is greater than the one-month average rate of change. **101.** The tangent to the graph is horizontal at that point, and so the graph is almost horizontal near that point.

103. Answers may vary.

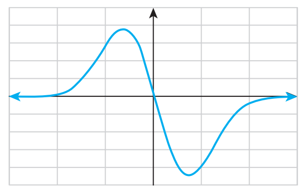


105. If $f(x) = mx + b$, then its average rate of change over any interval $[x, x + h]$ is $\dfrac{m(x+h) + b - (mx + b)}{h} = m$. Because this does not depend on h, the instantaneous rate is also equal to m. **107.** Increasing because the average rate of change appears to be rising as we get closer to 5 from the left (see the bottom row).

109. Answers may vary

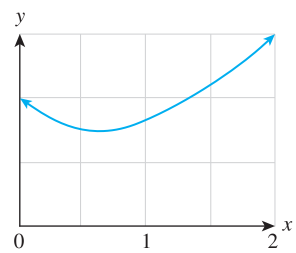


111. Answers may vary



113. (B)

115. Answers may vary.



Section 10.6

1. 4 **3.** 3 **5.** 7 **7.** 4 **9.** 14 **11.** 1 **13.** m **15.** $2x$ **17.** 3 **19.** $6x + 1$ **21.** $2 - 2x$ **23.** $3x^2 + 2$ **25.** $1/x^2$ **27.** m **29.** -1.2 **31.** 30.6 **33.** -7.1 **35.** 4.25 **37.** -0.6 **39.** $y = 4x - 7$ **41.** $y = -2x - 4$ **43.** $y = -3x - 1$ **45.** $s'(t) = -32t$; $s'(4) = -128$ ft/sec **47.** Annual U.S. imports from China were increasing by \$13.5 billion per year in 2000. **49.** $R'(t) = 34t + 100$. Annual U.S. sales of bottled water were increasing by 440 million gallons per year in 2000. **51.** $f'(8) = 26.6$ manatee deaths per 100,000 boats. At a level of 800,000 boats, the number of manatee deaths is increasing at a rate of 26.6 manatees per 100,000 additional boats. **53.** The algebraic method because it gives the exact value of the derivative. The other two approaches give only approximate values (except in some special cases). **55.** Because the algebraic computation of $f'(a)$ is exact and not an approximation, it makes no difference whether one uses the balanced difference quotient or the ordinary difference quotient in the algebraic computation. **57.** The computation results in a limit that cannot be evaluated.

Section 10.7

1. $5x^4$ **3.** $-4x^{-3}$ **5.** $-0.25x^{-0.75}$ **7.** $8x^3 + 9x^2$ **9.** $-1 - 1/x^2$

11. $\dfrac{dy}{dx} = 10(0) = 0$ (constant multiple and power rule)

13. $\dfrac{dy}{dx} = \dfrac{d}{dx}(x^2) + \dfrac{d}{dx}(x)$ (sum rule) $= 2x + 1$ (power rule)

15. $\dfrac{dy}{dx} = \dfrac{d}{dx}(4x^3) + \dfrac{d}{dx}(2x) - \dfrac{d}{dx}(1)$ (sum and difference)

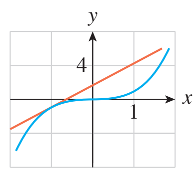
$= 4\dfrac{d}{dx}(x^3) + 2\dfrac{d}{dx}(x) - \dfrac{d}{dx}(1)$ (constant multiples)

$= 12x^2 + 2$ (power rule)

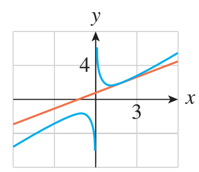
17. $f'(x) = 2x - 3$ **19.** $f'(x) = 1 + 0.5x^{-0.5}$

21. $g'(x) = -2x^{-3} + 3x^{-2}$ **23.** $g'(x) = -\frac{1}{x^2} + \frac{2}{x^3}$
25. $h'(x) = -\frac{0.8}{x^{1.4}}$ **27.** $h'(x) = -\frac{2}{x^3} - \frac{6}{x^4}$
29. $r'(x) = -\frac{2}{3x^2} + \frac{0.1}{2x^{1.1}}$ **31.** $r'(x) = \frac{2}{3} - \frac{0.1}{2x^{0.9}} - \frac{4.4}{3x^{2.1}}$
33. $t'(x) = |x|/x - 1/x^2$ **35.** $s'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$
37. $s'(x) = 3x^2$ **39.** $t'(x) = 1 - 4x$ **41.** $2.6x^{0.3} + 1.2x^{-2.2}$
43. $1.2(1 - |x|/x)$ **45.** $3at^2 - 4a$ **47.** $5.15x^{9.3} - 99x^{-2}$
49. $-\frac{2.31}{t^{2.1}} - \frac{0.3}{t^{0.4}}$ **51.** $4\pi r^2$ **53.** 3 **55.** −2 **57.** −5

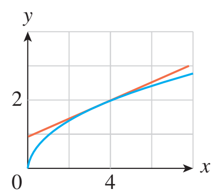
59. $y = 3x + 2$



61. $y = \frac{3}{4}x + 1$

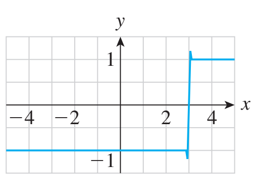


63. $y = \frac{1}{4}x + 1$



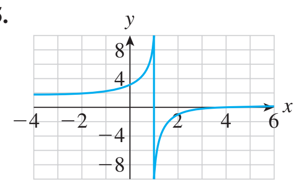
65. $x = -3/4$ **67.** No such values **69.** $x = 1, -1$

73.



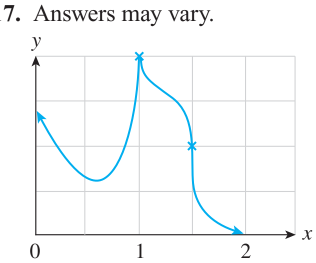
a. $x = 3$ **b.** None

75.



a. $x = 1$ **b.** $x = 4.2$

77. a. $f'(1) = 1/3$ **b.** Not differentiable at 0 **79. a.** Not differentiable at 1 **b.** Not differentiable at 0 **81.** Yes; 0 **83.** Yes; 12 **85.** No; 3 **87.** Yes; 3/2 **89.** Yes; Diverges to $-\infty$ **91.** Yes; Diverges to $-\infty$ **93. a.** $s'(t) = 3.04t + 9.45$ **b.** 52 teams/year **95.** $P'(t) = -5.2t + 13$; increasing at a rate of 2.6 percentage points per year **97.** 0.55 **99. a.** $s'(t) = -32t$; 0, −32, −64, −96, −128 ft/sec **b.** 5 seconds; downward at 160 ft/sec **101. a.** $E'(t) = 0.072t - 0.10$. In January 2004 the value of the euro was increasing at a rate of $0.188 per year. **b.** (D) **103. a.** $f'(x) = 7.1x - 30.2$ manatees per 100,000 boats. **b.** Increasing; the number of manatees killed per additional 100,000 boats increases as the number of boats increases. **c.** $f'(8) = 26.6$ manatees per 100,000 additional boats. At a level of 800,000 boats, the number of manatee deaths is increasing at a rate of 26.6 manatees per 100,000 additional boats. **105. a.** $c(t) - m(t)$ measures the combined market share of the other three providers (Comcast, Earthlink, and AOL); $c'(t) - m'(t)$ measures the rate of change of the combined market share of the other three providers. **b.** (A) **c.** (A) **d.** 3.72% per year. In 1992, the combined market share of the other three providers was increasing at a rate of about 3.72 percentage points per year. **107.** After graphing the curve $y = 3x^2$, draw the line passing through (−1, 3) with slope −6. **109.** The slope of the tangent line of g is twice the slope of the tangent line of f. **111.** $g'(x) = -f'(x)$ **113.** The left-hand side is not equal to the right-hand side. The *derivative* of the left-hand side is equal to the right-hand side, so your friend should have written $\frac{d}{dx}(3x^4 + 11x^5) = 12x^3 + 55x^4$ **115.** The derivative of a constant times a function is the constant times the derivative of the function, so that $f'(x) = (2)(2x) = 4x$. Your enemy mistakenly computed the *derivative* of the constant times the derivative of the function. (The derivative of a product of two functions is not the product of the derivative of the two functions. The rule for taking the derivative of a product is discussed in the next chapter.).
117. Answers may vary.



Section 10.8

1. $C'(1000) = \$4.80$ per item **3.** $C'(100) = \$99.90$ per item **5.** $C'(x) = 4$; $R'(x) = 8 - x/500$; $P'(x) = 4 - x/500$; $P'(x) = 0$ when $x = 2000$. Thus, at a production level of 2000, the profit is stationary (neither increasing nor decreasing) with respect to the production level. This may indicate a maximum profit at a production level of 2000. **7. a.** (B) **b.** (C) **c.** (C) **9. a.** $C'(x) = 2250 - 0.04x$. The cost is going up at a rate of $2,249,840 per television commercial. The exact cost of airing the fifth television commercial is $C(5) - C(4) = \$2{,}249{,}820$. **b.** $\bar{C}(x) = 150/x + 2250 - 0.02x$; $\bar{C}(4) = \$2{,}287{,}420$ per television commercial. The average cost of airing the first four television commercials is $2,287,420. **11. a.** $R'(x) = 0.90$, $P'(x) = 0.80 - 0.002x$ **b.** Revenue: $450, Profit: $80, Marginal revenue: $0.90, Marginal profit: −$0.20. The total revenue from the sale of 500 copies is $450. The profit from the production and sale of 500 copies is $80. Approximate revenue from the sale of the 501st copy is 90¢. Approximate loss from the sale of the 501st copy is 20¢. **c.** $x = 400$. The profit is a maximum when you produce and sell 400 copies. **13.** The profit on the sale of 1000 DVDs is $3000, and is decreasing at a rate of $3 per additional DVD sold. **15.** $P \approx \$257.07$ and $dP/dx \approx 5.07$. Your current profit is $257.07 per month, and this would increase at a rate of $5.07 per additional magazine in sales. **17. a.** $2.50 per pound **b.** $R(q) = 20{,}000/q^{0.5}$

c. $R(400) = \$1000$. This is the monthly revenue that will result from setting the price at \$2.50 per pound. $R'(400) = -\$1.25$ per pound of tuna. Thus, at a demand level of 400 pounds per month, the revenue is decreasing at a rate of \$1.25 per pound. **d.** The fishery should raise the price (to reduce the demand).
19. $P'(50) = \$350$. This means that, at an employment level of 50 workers, the firm's daily profit will increase at a rate of \$350 per additional worker it hires. **21. a.** (B) **b.** (B) **c.** (C)
23. a. $C(x) = 500{,}000 + 1{,}600{,}000x - 100{,}000\sqrt{x}$; $C'(x) = 1{,}600{,}000 - \dfrac{50{,}000}{\sqrt{x}}$; $\bar{C}(x) = \dfrac{500{,}000}{x} + 1{,}600{,}000 - \dfrac{100{,}000}{\sqrt{x}}$ **b.** $C'(3) \approx \$1{,}570{,}000$ per spot, $\bar{C}(3) \approx \$1{,}710{,}000$ per spot. The average cost will decrease as x increases. **25. a.** $C'(q) = 200q$; $C'(10) = \$2000$ per one-pound reduction in emissions. **b.** $S'(q) = 500$. Thus $S'(q) = C'(q)$ when $500 = 200q$, or $q = 2.5$ pounds per day reduction. **c.** $N(q) = C(q) - S(q) = 100q^2 - 500q + 4000$. This is a parabola with lowest point (vertex) given by $q = 2.5$. The net cost at this production level is $N(2.5) = \$3375$ per day. The value of q is the same as that for part (b). The net cost to the firm is minimized at the reduction level for which the cost of controlling emissions begins to increase faster than the subsidy. This is why we get the answer by setting these two rates of increase equal to each other. **27.** $M'(10) \approx 0.0002557$ mpg/mph. This means that, at a speed of 10 mph, the fuel economy is increasing at a rate of 0.0002557 miles per gallon per 1-mph increase in speed. $M'(60) = 0$ mpg/mph. This means that, at a speed of 60 mph, the fuel economy is neither increasing nor decreasing with increasing speed. $M'(70) \approx -0.00001799$. This means that, at 70 mph, the fuel economy is decreasing at a rate of 0.00001799 miles per gallon per 1-mph increase in speed. Thus 60 mph is the most fuel-efficient speed for the car. **29.** (C) **31.** (D) **33.** (B)
35. Cost is often measured as a function of the number of items x. Thus, $C(x)$ is the cost of producing (or purchasing, as the case may be) x items. **a.** The average cost function $\bar{C}(x)$ is given by $\bar{C}(x) = C(x)/x$. The marginal cost function is the derivative, $C'(x)$, of the cost function. **b.** The average cost $\bar{C}(r)$ is the slope of the line through the origin and the point on the graph where $x = r$. The marginal cost of the rth unit is the slope of the tangent to the graph of the cost function at the point where $x = r$. **c.** The average cost function $\bar{C}(x)$ gives the average cost of producing the first x items. The marginal cost function $C'(x)$ is the rate at which cost is changing with respect to the number of items x, or the incremental cost per item, and approximates the cost of producing the $(x + 1)$st item. **37.** The marginal cost **39.** Not necessarily. For example, it may be the case that the marginal cost of the 101st item is larger than the average cost of the first 100 items (even though the marginal cost is decreasing). Thus, adding this additional item will *raise* the average cost. **41.** The circumstances described suggest that the average cost function is at a relatively low point at the current production level, and so it would be appropriate to advise the company to maintain current production levels; raising or lowering the production level will result in increasing average costs.

Chapter 10 Review

1. 5 **3.** Does not exist **5. a.** −1 **b.** 3 **c.** Does not exist
7. −4/5 **9.** −1 **11.** Diverges to $-\infty$

13.

h	1	0.01	0.001
Ave. Rate of Change	−0.5	−0.9901	−0.9990

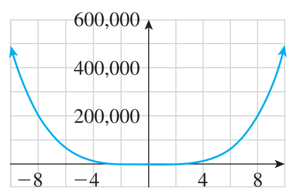
Slope ≈ -1

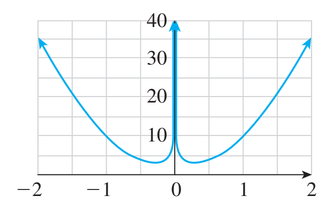
15.

h	1	0.01	0.001
Avg. Rate of Change	6.3891	2.0201	2.0020

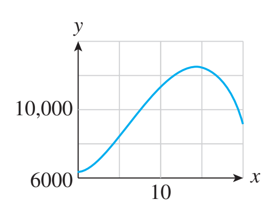
Slope ≈ 2

17. a. (i) P **(ii)** Q **(iii)** R **(iv)** S **19. (i)** Q **(ii)** None **(iii)** None **(iv)** None **21. a.** (B) **b.** (B) **c.** (B) **d.** (A) **e.** (C) **23.** $2x + 1$ **25.** $2/x^2$ **27.** $50x^4 + 2x^3 - 1$
29. $9x^2 + x^{-2/3}$ **31.** $1 - 2/x^3$
33. $-4/(3x^2) + 0.2/x^{1.1} + 1.1x^{0.1}/3.2$
35. `50*x^4+2*x^3-1` **37.** `9*x^2+1/(x^2)^(1/3)`





39. a. $P(3) = 25$: O'Hagan purchased the stock at \$25. $\lim_{t\to 3^-} P(t) = 25$: The value of the stock had been approaching \$25 up to the time he bought it. $\lim_{t\to 3^+} P(t) = 10$: The value of the stock dropped to \$10 immediately after he bought it.
b. Continuous but not differentiable. Interpretation: the stock price changed continuously but suddenly reversed direction (and started to go up) the instant O'Hagan sold it. **41. a.** 500 books per week **b.** [3, 4], [4, 5] **c.** [3, 5]; 650 books per week
43. a. 274 books per week **b.** 636 books per week **c.** No; the function w begins to decrease after $t = 14$. Graph:



45. a. \$2.88 per book **b.** \$3.715 per book **c.** Approximately −\$0.000104 per book, per additional book sold. **d.** At a sales level of 8000 books per week, the cost is increasing at a rate of \$2.88 per book (so that the 8001st book costs approximately \$2.88 to sell), and it costs an average of \$3.715 per book to sell the first 8000 books. Moreover, the average cost is decreasing at a rate of \$0.000104 per book, per additional book sold.

Chapter 11

Section 11.1

1. 3 **3.** $3x^2$ **5.** $2x+3$ **7.** $210x^{1.1}$ **9.** $-2/x^2$ **11.** $2x/3$ **13.** $3(4x^2-1)+3x(8x)=36x^2-3$ **15.** $3x^2(1-x^2)+x^3(-2x)=3x^2-5x^4$ **17.** $2(2x+3)+(2x+3)(2)=8x+12$ **19.** $3\sqrt{x}/2$ **21.** $(x^2-1)+2x(x+1)=(x+1)(3x-1)$ **23.** $(x^{-0.5}+4)(x-x^{-1})+(2x^{0.5}+4x-5)(1+x^{-2})$ **25.** $8(2x^2-4x+1)(x-1)$ **27.** $(1/3.2-3.2/x^2)(x^2+1)+2x(x/3.2+3.2/x)$ **29.** $2x(2x+3)(7x+2)+2x^2(7x+2)+7x^2(2x+3)$ **31.** $5.3(1-x^{2.1})(x^{-2.3}-3.4)-2.1x^{1.1}(5.3x-1)\cdot(x^{-2.3}-3.4)-2.3x^{-3.3}(5.3x-1)(1-x^{2.1})$

33. $\dfrac{1}{2\sqrt{x}}\left(\sqrt{x}+\dfrac{1}{x^2}\right)+(\sqrt{x}+1)\left(\dfrac{1}{2\sqrt{x}}-\dfrac{2}{x^3}\right)$

35. $\dfrac{2(3x-1)-3(2x+4)}{(3x-1)^2}=-14/(3x-1)^2$

37. $\dfrac{(4x+4)(3x-1)-3(2x^2+4x+1)}{(3x-1)^2}=(6x^2-4x-7)/(3x-1)^2$

39. $\dfrac{(2x-4)(x^2+x+1)-(x^2-4x+1)(2x+1)}{(x^2+x+1)^2}=(5x^2-5)/(x^2+x+1)^2$

41. $\dfrac{(0.23x^{-0.77}-5.7)(1-x^{-2.9})-2.9x^{-3.9}(x^{0.23}-5.7x)}{(1-x^{-2.9})^2}$

43. $\dfrac{\frac{1}{2}x^{-1/2}(x^{1/2}-1)-\frac{1}{2}x^{-1/2}(x^{1/2}+1)}{(x^{1/2}-1)^2}=\dfrac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$

45. $-3/x^4$

47. $\dfrac{[(x+1)+(x+3)](3x-1)-3(x+3)(x+1)}{(3x-1)^2}=(3x^2-2x-13)/(3x-1)^2$

49. $\dfrac{[(x+1)(x+2)+(x+3)(x+2)+(x+3)(x+1)](3x-1)-3(x+3)(x+1)(x+2)}{(3x-1)^2}$

51. $4x^3-12x^2+2x-480$ **53.** $1+2/(x+1)^2$ **55.** $2x-1-2/(x+1)^2$ **57.** $4x^3-2x$ **59.** 64 **61.** 3 **63.** $y=12x-8$ **65.** $y=x/4+1/2$ **67.** $y=-2$ **69.** $q'(5)=1000$ units/month (sales are increasing at a rate of 1000 units per month); $p'(5)=-\$10$/month (the price of a sound system is dropping at a rate of \$10 per month); $R'(5)=900{,}000$ (revenue is increasing at a rate of \$900,000 per month) **71.** \$242 million; increasing at a rate of \$39 million per year **73.** Decreasing at a rate of \$1 per day **75.** Decreasing at a rate of approximately \$0.10 per month

77. $M'(x)=\dfrac{3000(3600x^{-2}-1)}{(x+3600x^{-1})^2}$; $M'(10)\approx 0.7670$ mpg/mph. This means that, at a speed of 10 mph, the fuel economy is increasing at a rate of 0.7670 miles per gallon per one mph increase in speed. $M'(60)=0$ mpg/mph. This means that, at a speed of 60 mph, the fuel economy is neither increasing nor decreasing with increasing speed. $M'(70)\approx -0.0540$. This means that, at 70 mph, the fuel economy is decreasing at a rate of 0.0540 miles per gallon per one mph increase in speed. 60 mph is the most fuel-efficient speed for the car. (In the next chapter we shall discuss how to locate largest values in general.) **79.** Increasing at a rate of about \$3420 million per year.

81. $R'(p)=-\dfrac{5.625}{(1+0.125p)^2}$; $R'(4)=-2.5$ thousand organisms per hour, per 1000 organisms. This means that the reproduction rate of organisms in a culture containing 4000 organisms is declining at a rate of 2500 organisms per hour, per 1000 additional organisms. **83.** Oxygen consumption is decreasing at a rate of 1600 milliliters per day. This is due to the fact that the number of eggs is decreasing, because $C'(25)$ is positive. **85. a.** $c(t)-m(t)$ represents the combined market share of the other three providers (Comcast, Earthlink, and AOL). $m(t)/c(t)$ represents MSN's market share as a fraction of the four providers considered. **b.** $\dfrac{d}{dt}\left(\dfrac{m(t)}{c(t)}\right)\Big|_{t=3}\approx -0.043$ (or -4.3 percentage points) per year. In June, 2003, MSN's market share as a fraction of the four providers considered was decreasing at a rate of about 0.043 (or 4.3 percentage points) per year. **87.** The analysis is suspect, because it seems to be asserting that the annual increase in revenue, which we can think of as dR/dt, is the product of the annual increases, dp/dt in price, and dq/dt in sales. However, because $R=pq$, the product rule implies that dR/dt is not the product of dp/dt and dq/dt, but is instead $\dfrac{dR}{dt}=\dfrac{dp}{dt}\cdot q+p\cdot\dfrac{dq}{dt}$. **89.** Answers will vary; $q=-p+1000$ is one example. **91.** Mine; it is increasing twice as fast as yours. The rate of change of revenue is given by $R'(t)=p'(t)q(t)$ because $q'(t)=0$ Thus, $R'(t)$ does not depend on the selling price $p(t)$. **93.** (A)

Section 11.2

1. $4(2x+1)$ **3.** $-(x-1)^{-2}$ **5.** $2(2-x)^{-3}$ **7.** $(2x+1)^{-0.5}$ **9.** $-4(4x-1)^{-2}$ **11.** $-3/(3x-1)^2$ **13.** $4(x^2+2x)^3(2x+2)$ **15.** $-4x(2x^2-2)^{-2}$ **17.** $-5(2x-3)(x^2-3x-1)^{-6}$ **19.** $-6x/(x^2+1)^4$ **21.** $1.5(0.2x-4.2)(0.1x^2-4.2x+9.5)^{0.5}$ **23.** $4(2s-0.5s^{-0.5})(s^2-s^{0.5})^3$ **25.** $-x/\sqrt{1-x^2}$ **27.** $-[(x+1)(x^2-1)]^{-3/2}(3x-1)(x+1)$ **29.** $6.2(3.1x-2)+6.2/(3.1x-2)^3$ **31.** $2[(6.4x-1)^2+(5.4x-2)^3][12.8(6.4x-1)+16.2(5.4x-2)^2]$ **33.** $-2(x^2-3x)^{-3}(2x-3)(1-x^2)^{0.5}-x(x^2-3x)^{-2}(1-x^2)^{-0.5}$ **35.** $-56(x+2)/(3x-1)^3$ **37.** $3z^2(1-z^2)/(1+z^2)^4$ **39.** $3[(1+2x)^4-(1-x)^2]^2[8(1+2x)^3+2(1-x)]$ **41.** $-0.43(x+1)^{-1.1}[2+(x+1)^{-0.1}]^{3.3}$

43. $-\dfrac{\left(\frac{1}{\sqrt{2x+1}}-2x\right)}{\left(\sqrt{2x+1}-x^2\right)^2}$

45. $54(1+2x)^2\left(1+(1+2x)^3\right)^2\left(1+\left(1+(1+2x)^3\right)^3\right)^2$

47. $(100x^{99}-99x^{-2})dx/dt$ **49.** $(-3r^{-4}+0.5r^{-0.5})dr/dt$

51. $4\pi r^2dr/dt$ **53.** $-47/4$ **55.** $1/3$ **57.** $-5/3$ **59.** $1/4$ **61.** $y=35(7+0.2t)^{-0.25}$; -0.11 percentage points per month.

63. $\left.\dfrac{dP}{dn}\right|_{n=10} = 146{,}454.9$. At an employment level of 10 engineers, Paramount will increase its profit at a rate of \$146,454.90 per additional engineer hired. **65.** −\$30 per additional ruby sold. The revenue is decreasing at a rate of \$30 per additional ruby sold. **67.** $\dfrac{dy}{dt} = \dfrac{dy}{dx}\dfrac{dx}{dt} = (1.5)(-2) = -3$ murders per 100,000 residents/yr each year. **69.** 0.000158 manatees per boat, or 15.8 manatees per 100,000 boats. Approximately 15.8 more manatees are killed each year for each additional 100,000 registered boats. **71.** 12π mi^2/h **73.** $\$200{,}000\pi$/week $\approx$ \$628,000/week **75. a.** $q'(4) \approx 333$ units per month **b.** $dR/dq = \$800$/unit **c.** $dR/dt \approx \$267{,}000$ per month **77.** 3% per year **79.** 8% per year **81.** The glob squared, times the derivative of the glob. **83.** The derivative of a quantity cubed is three times the (original) quantity squared, times the derivative of the quantity. Thus, the correct answer is $3(3x^3 - x)^2(9x^2 - 1)$. **85.** Following the calculation thought experiment, pretend that you are evaluating the function at a specific value of x. If the last operation you would perform is addition or subtraction, look at each summand separately. If the last operation is multiplication, use the product rule first; if it is division, use the quotient rule first; if it is any other operation (such as raising a quantity to a power or taking a radical of a quantity), then use the chain rule first. **87.** An example is

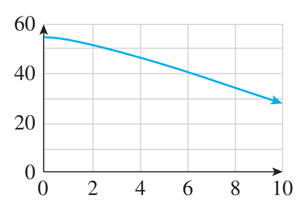
$$f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + 1}}}}}.$$

Section 11.3

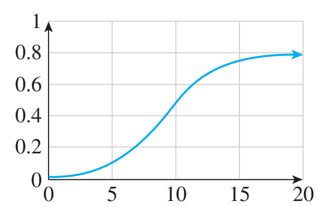
1. $1/(x-1)$ **3.** $1/(x \ln 2)$ **5.** $2x/(x^2+3)$ **7.** e^{x+3}
9. $-e^{-x}$ **11.** $4^x \ln 4$ **13.** $2^{x^2-1}\, 2x \ln 2$ **15.** $1 + \ln x$
17. $2x \ln x + (x^2+1)/x$ **19.** $10x(x^2+1)^4 \ln x + (x^2+1)^5/x$
21. $3/(3x-1)$ **23.** $4x/(2x^2+1)$
25. $(2x - 0.63x^{-0.7})/(x^2 - 2.1x^{0.3})$
27. $-2/(-2x+1) + 1/(x+1)$ **29.** $3/(3x+1) - 4/(4x-2)$
31. $1/(x+1) + 1/(x-3) - 2/(2x+9)$ **33.** $5.2/(4x-2)$
35. $2/(x+1) - 9/(3x-4) - 1/(x-9)$ **37.** $\dfrac{1}{(x+1)\ln 2}$
39. $\dfrac{1 - 1/t^2}{(t + 1/t)\ln 3}$ **41.** $\dfrac{2\ln|x|}{x}$ **43.** $\dfrac{2}{x} - \dfrac{2\ln(x-1)}{x-1}$
45. $e^x(1+x)$ **47.** $1/(x+1) + 3e^x(x^3+3x^2)$
49. $e^x(\ln|x| + 1/x)$ **51.** $2e^{2x+1}$ **53.** $(2x-1)e^{x^2-x+1}$
55. $2xe^{2x-1}(1+x)$ **57.** $4(e^{2x-1})^2$ **59.** $2 \cdot 3^{2x-4} \ln 3$
61. $2 \cdot 3^{2x+1} \ln 3 + 3e^{3x+1}$ **63.** $\dfrac{2x3^{x^2}[(x^2+1)\ln 3 - 1]}{(x^2+1)^2}$
65. $-4/(e^x - e^{-x})^2$ **67.** $5e^{5x-3}$ **69.** $-\dfrac{\ln x + 1}{(x \ln x)^2}$
71. $2(x-1)$ **73.** $\dfrac{1}{x \ln x}$ **75.** $\dfrac{1}{2x \ln x}$
77. $y = (e/\ln 2)(x-1) \approx 3.92(x-1)$ **79.** $y = x$
81. $y = -[1/(2e)](x-1) + e$ **83.** Average price: \$1.4 million; increasing at a rate of about \$220,000 per year. **85.** \$451.00 per year **87.** \$446.02 per year **89.** 300 articles per year **91.** 3,300,000 cases/week; 11,000,000 cases/week; 640,000 cases/week **93.** 310 articles per year **95.** 277,000 people/yr **97.** 0.000283 g/yr **99. a.** (A) **b.** The verbal SAT increases by approximately 1 point. **c.** $S'(x)$ decreases with increasing x, so that as parental income increases, the effect on SAT scores decreases. **101. a.** −6.25 years/child; When the fertility rate is 2 children per woman, the average age of a population is dropping at a rate of 6.25 years per one-child increase in the fertility rate. **b.** 0.160
103. a. $W'(t) = -\dfrac{1500(0.77)(1.16)^{-t}(-1)\ln(1.16)}{(1+0.77(1.16)^{-t})^2}$

$$\approx \frac{171.425(1.16)^{-t}}{(1+0.77(1.16)^{-t})^2};\ W'(6) \approx 40.624 \approx 41$$

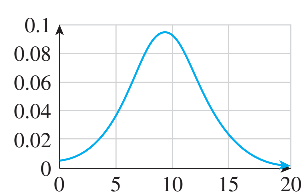
to two significant digits. The constants in the model are specified to two and three significant digits, so we cannot expect the answer to be accurate to more than two digits. In other words, all digits from the third on are probably meaningless. The answer tells one that in 1996, the number of authorized wiretaps was increasing at a rate of approximately 41 wiretaps per year.
b. (A) Graph:



105. a.



$p'(10) \approx 0.09$, so the percentage of firms using numeric control is increasing at a rate of 9 percentage points per year after 10 years. **b.** 0.80. Thus, in the long run, 80% of all firms will be using numeric control.
c. $p'(t) = 0.3816e^{4.46-0.477t}/(1+e^{4.46-0.477t})^2$.
$p'(10) = 0.0931$. Graph:



d. 0. Thus, in the long run, the percentage of firms using numeric control will stop increasing.
107. $R(t) = 350e^{-0.1t}(39t+68)$ million dollars; $R(2) \approx$ \$42 billion; $R'(2) \approx$ \$7 billion per year **109.** e raised to the glob, times the derivative of the glob. **111.** 2 raised to the glob, times the derivative of the glob, times the natural logarithm of 2.
113. The power rule does not apply when the exponent is not constant. The derivative of 3 raised to a quantity is 3 raised to the quantity, times the derivative of the quantity, times ln 3. Thus, the correct answer is $3^{2x} 2 \ln 3$. **115.** No. If $N(t)$ is exponential, so is its derivative. **117.** If $f(x) = e^{kx}$, then the fractional rate of change is $\dfrac{f'(x)}{f(x)} = \dfrac{ke^{kx}}{e^{kx}} = k$, the fractional growth rate. **119.** If $A(t)$ is growing exponentially, then $A(t) = A_0e^{kt}$ for constants A_0 and k. Its percentage rate of change is then

$$\frac{A'(t)}{A(t)} = \frac{kA_0e^{kt}}{A_0e^{kt}} = k,\ \text{a constant.}$$

Section 11.4

1. $-2/3$ **3.** x **5.** $(y-2)/(3-x)$ **7.** $-y$

9. $-\dfrac{y}{x(1+\ln x)}$ **11.** $-x/y$ **13.** $-2xy/(x^2-2y)$

15. $-(6+9x^2y)/(9x^3-x^2)$ **17.** $3y/x$

19. $(p+10p^2q)/(2p-q-10pq^2)$

21. $(ye^x-e^y)/(xe^y-e^x)$ **23.** $se^{st}/(2s-te^{st})$

25. $ye^x/(2e^x+y^3e^y)$ **27.** $(y-y^2)/(-1+3y-y^2)$

29. $-y/(x+2y-xye^y-y^2e^y)$ **31. a.** 1 **b.** $y=x-3$

33. a. -2 **b.** $y=-2x$ **35. a.** -1 **b.** $y=-x+1$

37. a. -2000 **b.** $y=-2000x+6000$ **39. a.** 0 **b.** $y=1$

41. a. -0.1898 **b.** $y=-0.1898x+1.4721$

43. $\dfrac{2x+1}{4x-2}\left[\dfrac{2}{2x+1}-\dfrac{4}{4x-2}\right]$

45. $\dfrac{(3x+1)^2}{4x(2x-1)^3}\left[\dfrac{6}{3x+1}-\dfrac{1}{x}-\dfrac{6}{2x-1}\right]$

47. $(8x-1)^{1/3}(x-1)\left[\dfrac{8}{3(8x-1)}+\dfrac{1}{x-1}\right]$

49. $(x^3+x)\sqrt{x^3+2}\left[\dfrac{3x^2+1}{x^3+x}+\dfrac{1}{2}\dfrac{3x^2}{x^3+2}\right]$

51. $x^x(1+\ln x)$ **53.** $-\$3000$ per worker. The monthly budget to maintain production at the fixed level P is decreasing by approximately \$3000 per additional worker at an employment level of 100 workers and a monthly operating budget of \$200,000.

55. -125 T-shirts per dollar; when the price is set at \$5, the demand is dropping by 125 T-shirts per \$1 increase in price.

57. $\left.\dfrac{dk}{de}\right|_{e=15}=-0.307$ carpenters per electrician. This means that, for a \$200,000 house whose construction employs 15 electricians, adding one more electrician would cost as much as approximately 0.307 additional carpenters. In other words, one electrician is worth approximately 0.307 carpenters.

59. a. 22.93 hours. (The other root is rejected because it is larger than 30.) **b.** $\dfrac{dt}{dx}=\dfrac{4t-20x}{0.4t-4x}$; $\left.\dfrac{dt}{dx}\right|_{x=3.0}\approx-11.2$ hours per grade point. This means that, for a 3.0 student who scores 80 on the examination, 1 grade point is worth approximately 11.2 hours.

61. $\dfrac{dr}{dy}=2\dfrac{r}{y}$, so $\dfrac{dr}{dt}=2\dfrac{r}{y}\dfrac{dy}{dt}$ by the chain rule.

63. x, y, y, x

65. Then $\ln y=\ln f(x)+\ln g(x)$, and $\dfrac{1}{y}\dfrac{dy}{dx}=\dfrac{f'(x)}{f(x)}+\dfrac{g'(x)}{g(x)}$, so $\dfrac{dy}{dx}=y\left(\dfrac{f'(x)}{f(x)}+\dfrac{g'(x)}{g(x)}\right)=f(x)g(x)\left(\dfrac{f'(x)}{f(x)}+\dfrac{g'(x)}{g(x)}\right)=f'(x)g(x)+f(x)g'(x)$.

67. Writing $y=f(x)$ specifies y as an explicit function of x. This can be regarded as an equation giving y as an *implicit* function of x. The procedure of finding dy/dx by implicit differentiation is then the same as finding the derivative of y as an explicit function of x: we take d/dx of both sides. **69.** Differentiate both sides of the equation $y=f(x)$ with respect to y to get $1=f'(x)\cdot\dfrac{dx}{dy}$, giving $\dfrac{dx}{dy}=\dfrac{1}{f'(x)}=\dfrac{1}{dy/dx}$.

Chapter 11 Review

1. $e^x(x^2+2x-1)$ **3.** $20x(x^2-1)^9$

5. $e^x(x^2+1)^9(x^2+20x+1)$

7. $3^x[(x-1)\ln 3-1]/(x-1)^2$ **9.** $2xe^{x^2-1}$

11. $2x/(x^2-1)$ **13.** $x=(1-\ln 2)/2$

15. None **17.** $\dfrac{2x-1}{2y}$ **19.** $-y/x$

21. $\dfrac{(2x-1)^4(3x+4)}{(x+1)(3x-1)^3}\left[\dfrac{8}{2x-1}+\dfrac{3}{3x+4}-\dfrac{1}{x+1}-\dfrac{9}{3x-1}\right]$

23. $y=x+2$

25. $R'(0)=p'(0)q(0)+p(0)q'(0)$
$=(-1)(1000)+20(200)=\$3000$ per week (rising)

27. $R=pq$ gives $R'=p'q+pq'$. Thus, $R'/R=R'/(pq)=(p'q+pq')/pq=p'/p+q'/q$

29. \$110 per year **31.** $s'(t)=\dfrac{2460.7e^{-0.55(t-4.8)}}{(1+e^{-0.55(t-4.8)})^2}$; 553 books per week **33.** 616.8 hits per day per week

Chapter 12

Section 12.1

1. Absolute min.: $(-3,-1)$, relative max: $(-1,1)$, relative min: $(1,0)$, absolute max: $(3,2)$ **3.** Absolute min: $(3,-1)$ and $(-3,-1)$, absolute max: $(1,2)$ **5.** Absolute min: $(-3,0)$ and $(1,0)$, absolute max: $(-1,2)$ and $(3,2)$ **7.** Relative min: $(-1,1)$ **9.** Absolute min: $(-3,-1)$, relative max: $(-2,2)$, relative min: $(1,0)$, absolute max: $(3,3)$ **11.** Relative max: $(-3,0)$, absolute min: $(-2,-1)$, stationary non-extreme point: $(1,1)$ **13.** Absolute max: $(0,1)$, absolute min: $(2,-3)$, relative max: $(3,-2)$ **15.** Absolute min: $(-4,-16)$, absolute max: $(-2,16)$, absolute min: $(2,-16)$, absolute max: $(4,16)$

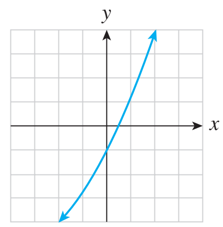
17. Absolute min: $(-2,-10)$, absolute max: $(2,10)$

19. Absolute min: $(-2,-4)$, relative max: $(-1,1)$, relative min: $(0,0)$ **21.** Relative max: $(-1,5)$, absolute min: $(3,-27)$

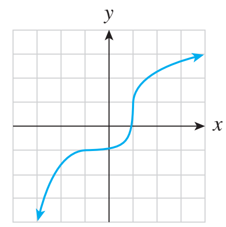
23. Absolute min: $(0,0)$ **25.** Absolute maxima at $(0,1)$ and $(2,1)$, absolute min at $(1,0)$ **27.** Relative maximum at $(-2,-1/3)$, relative minimum at $(-1,-2/3)$, absolute maximum at $(0,1)$ **29.** Relative min: $(-2,5/3)$, relative max: $(0,-1)$, relative min: $(2,5/3)$ **31.** Relative max: $(0,0)$; absolute min: $(1/3,-2\sqrt{3}/9)$ **33.** Relative max: $(0,0)$, absolute min: $(1,-3)$ **35.** No relative extrema **37.** Absolute min: $(1,1)$

39. Relative max: $(-1,1+1/e)$, absolute min: $(0,1)$, absolute max: $(1,e-1)$ **41.** Relative max: $(-6,-24)$, relative min: $(-2,-8)$ **43.** Absolute max $(1/\sqrt{2},\sqrt{e/2})$, absolute min: $(-1/\sqrt{2},-\sqrt{e/2})$ **45.** Relative min at $(0.15,-0.52)$ and $(2.45,8.22)$, relative max at $(1.40,0.29)$ **47.** Absolute max at $(-5,700)$, relative max at $(3.10,28.19)$ and $(6,40)$, absolute min at $(-2.10,-392.69)$ and relative min at $(5,0)$. **49.** Stationary minimum at $x=-1$ **51.** Stationary minima at $x=-2$ and $x=2$, stationary maximum at $x=0$ **53.** Singular minimum at $x=0$, stationary non-extreme point at $x=1$ **55.** Stationary minimum at $x=-2$, singular non-extreme points at $x=-1$ and $x=1$, stationary maximum at $x=2$

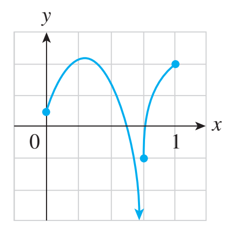
57. Answers will vary.



59. Answers will vary.

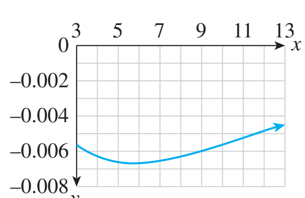


61. Not necessarily; it could be neither a relative maximum nor a relative minimum, as in the graph of $y = x^3$ at the origin.
63. Answers will vary.



Section 12.2

1. $x = y = 5$; $P = 25$ **3.** $x = y = 3$; $S = 6$ **5.** $x = 2$, $y = 4$; $F = 20$ **7.** $x = 20$, $y = 10$, $z = 20$; $P = 4000$ **9.** 5×5 **11.** 5000 MP3 players, giving an average cost of \$30 per MP3 player **13.** $\sqrt{40} \approx 6.32$ pounds of pollutant per day, for an average cost of about \$1265 per pound **15.** 2.5 lb **17.** $5 \times 10 = 50$ square feet **19.** \$10 **21.** \$30 **23. a.** \$1.41 per pound **b.** 5000 pounds **c.** \$7071.07 per month **25.** 34.5¢ per pound, for an annual (per capita) revenue of \$5.95 **27.** \$42.50 per ruby, for a weekly profit of \$408.33 **29. a.** 656 headsets, for a profit of \$28,120 **b.** \$143 per headset **31.** $13\frac{1}{3}$ in $\times 3\frac{1}{3}$ in $\times 1\frac{1}{3}$ in for a volume of $1600/27 \approx$ 59 cubic inches **33.** $5 \times 5 \times 5$ cm **35.** $l = w = h \approx 20.67$ in, volume ≈ 8827 in^3 **37.** $l = 30$ in, $w = 15$ in, $h = 30$ in **39.** $l = 36$ in, $w = h = 18$ in, $V = 11,664$ in^3 **41. a.** 1.6 years, or year 2001.6; **b.** $R_{max} = \$28,241$ million **43.** $t = 2.5$ or midway through 1972.; $D(2.5)/S(2.5) \approx 4.09$ The number of new (approved) drugs per \$1 billion of spending on research and development reached a high of around 4 approved drugs per \$1 billion midway through 1972. **45.** 30 years from now **47.** 55 days **49.** 1600 copies. At this value of x, average profit equals marginal profit; beyond this the marginal profit is smaller than the average. **51.** Increasing most rapidly in 1992; increasing least rapidly in 1980 **53.** Maximum when $t = 17$ days. This means that the embryo's oxygen consumption is increasing most rapidly 17 days after the egg is laid. **55.** $h = r \approx 11.7$ cm **57.** 25 additional trees **59.** 71 employees **61.** Fourth quarter of 2003 ($t \approx 3.7$); 160 thousand iPods per quarter
63. Graph of derivative:

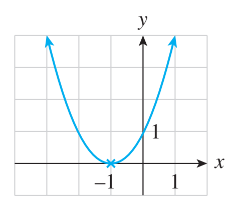


Absolute minimum occurs at approximately $t = 6$, with value approximately -0.0067. The fraction of bottled water sales due to sparkling water was decreasing most rapidly in 1996. At that time it was decreasing at a rate of 0.67 percentage points per year. **65.** You should sell them in 17 years' time, when they will be worth approximately \$3960. **67.** (D) **69.** The problem is uninteresting because the company can accomplish the objective by cutting away the entire sheet of cardboard, resulting in a box with surface area zero. **71.** Not all absolute extrema occur at stationary points; some may occur at an endpoint or singular point of the domain, as in Exercises 23, 24, 51 and 52. **73.** The minimum of dq/dp is the fastest that the demand is dropping in response to increasing price.

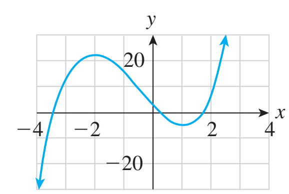
Section 12.3

1. 6 **3.** $4/x^3$ **5.** $-0.96x^{-1.6}$ **7.** $e^{-(x-1)}$ **9.** $2/x^3 + 1/x^2$
11. a. $a = -32$ ft/sec^2 **b.** $a = -32$ ft/sec^2
13. a. $a = 2/t^3 + 6/t^4$ ft/sec^2 **b.** $a = 8$ ft/sec^2
15. a. $a = -1/(4t^{3/2}) + 2$ ft/sec^2
b. $a = 63/32$ ft/sec^2 **17.** (1, 0) **19.** (1, 0) **21.** None
23. $(-1, 0)$, (1, 1) **25.** Points of inflection at $x = -1$ and $x = 1$ **27.** One point of inflection, at $x = -2$ **29.** Points of inflection at $x = -2, x = 0, x = 2$ **31.** Points of inflection at $x = -2$ and $x = 2$

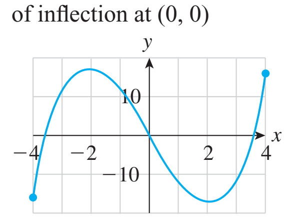
33. Absolute min at $(-1, 0)$; no points of inflection



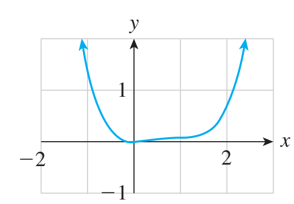
35. Relative max at $(-2, 21)$; relative min at $(1, -6)$; point of inflection at $(-1/2, 15/2)$



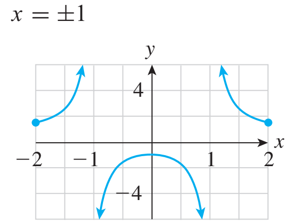
37. Absolute min at $(-4, -16)$ and $(2, -16)$; absolute max at $(-2, 16)$ and $(4, 16)$; point of inflection at (0, 0)



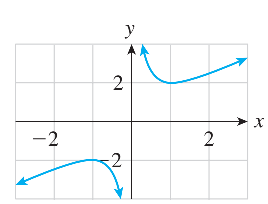
39. Absolute min at (0, 0); points of inflection at (1/3, 11/324) and (1, 1/12)



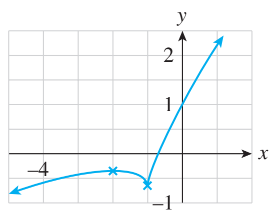
41. Relative min at $(-2, 5/3)$ and $(2, 5/3)$; relative max at $(0, -1)$; vertical asymptotes: $x = \pm 1$



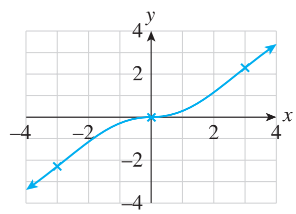
43. Relative min at (1, 2); relative max at $(-1, -2)$; vertical asymptote: $y = 0$



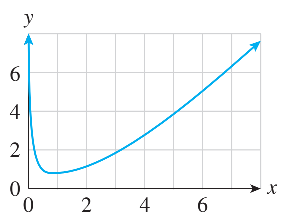
45. Relative maximum at $(-2, -1/3)$; relative minimum at $(-1, -2/3)$; no points of inflection



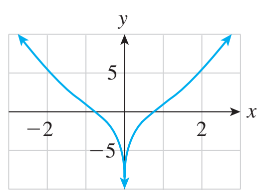
47. No extrema; points of inflection at $(0, 0)$, $(-3, -9/4)$, and $(3, 9/4)$



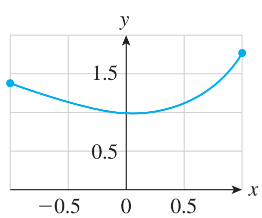
49. Absolute min at $(1, 1)$; vertical asymptote at $x = 0$



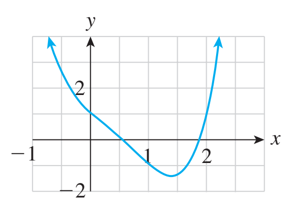
51. No relative extrema; point of inflection at $(1, 1)$ and $(-1, 1)$; vertical asymptote at $x = 0$



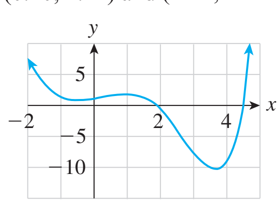
53. Absolute min at $(0, 1)$, absolute max at $(1, e - 1)$, relative max at $(-1, 1 + e^{-1})$



55. Absolute min at $(1.40, -1.49)$; points of inflection: $(0.21, 0.61)$, $(0.79, -0.55)$

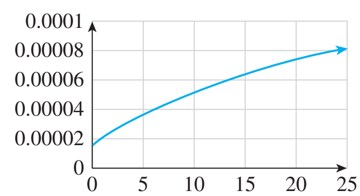


57. Relative min at $(-0.46, 0.73)$; relative max at $(0.91, 1.73)$; absolute min at $(3.73, -10.22)$; points of inflection at $(0.20, 1.22)$ and $(2.83, -5.74)$



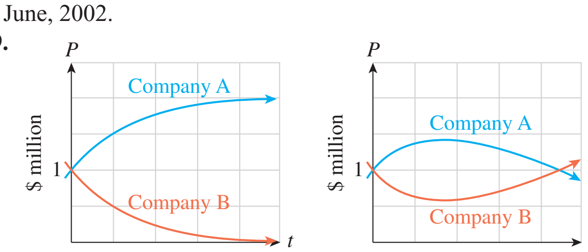
59. -3.8 m/s^2 **61.** $6t - 2$ ft/s^2; increasing **63.** Accelerating by 34 million gals/yr^2 **65. a.** 400 ml **b.** 36 ml/day **c.** -1 ml/day^2 **67. a.** Two years into the epidemic **b.** Two years into the epidemic **69. a.** 2000 **b.** 2002 **c.** 1998 **71.** Concave up for $8 < t < 20$, concave down for $0 < t < 8$, point of inflection around $t = 8$. The percentage of articles written by researchers in the U.S. was decreasing most rapidly at around $t = 8$ (1991). **73. a.** (B) **b.** (B) **c.** (A) **75. a.** There are no points of inflection in the graph of S. **b.** Because the graph is concave up, the derivative of S is increasing, and so the rate of *decrease* of SAT scores with increasing numbers of prisoners is diminishing. In other words, the apparent effect of more prisoners is diminishing.

77. a. $\left.\dfrac{d^2n}{ds^2}\right|_{s=3} = -21.494$. Thus, for a firm with annual sales of \$3 million, the rate at which new patents are produced decreases with increasing firm size. This means that the returns (as measured in the number of new patents per increase of \$1 million in sales) are diminishing as the firm size increases. **b.** $\left.\dfrac{d^2n}{ds^2}\right|_{s=7} = 13.474$. Thus, for a firm with annual sales of \$7 million, the rate at which new patents are produced increases with increasing firm size by 13.474 new patents per \$1 million increase in annual sales. **c.** There is a point of inflection when $s \approx 5.4587$, so that in a firm with sales of \$5,458,700 per year, the number of new patents produced per additional \$1 million in sales is a minimum. **79.** Concave down; (C). Graph:



81. About \$570 per year, after about 12 years **83.** Increasing most rapidly in 17.64 years, decreasing most rapidly now (at $t = 0$) **85.** Nonnegative **87.** Daily sales were decreasing most rapidly in June, 2002.

89.



91. At a point of inflection, the graph of a function changes either from concave up to concave down, or vice versa. If it changes from concave up to concave down, then the derivative changes from increasing to decreasing, and hence has a relative maximum. Similarly, if it changes from concave down to concave up, the derivative has a relative minimum.

Section 12.4

1. $P = 10{,}000$; $\dfrac{dP}{dt} = 1000$ **3.** Let R be the annual revenue of my company, and let q be annual sales. $R = 7000$ and $\dfrac{dR}{dt} = -700$. Find $\dfrac{dq}{dt}$. **5.** Let p be the price of a pair of shoes, and let q be the demand for shoes. $\dfrac{dp}{dt} = 5$. Find $\dfrac{dq}{dt}$. **7.** Let T be the average global temperature, and let q be the number of Bermuda shorts sold per year. $T = 60$ and $\dfrac{dT}{dt} = 0.1$. Find $\dfrac{dq}{dt}$. **9. a.** $6/(100\pi) \approx 0.019$ km/sec **b.** $6/(8\sqrt{\pi}) \approx 0.4231$ km/sec **11.** $3/(4\pi) \approx 0.24$ ft/min

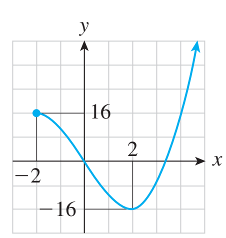
13. 7.5 ft/sec **15.** Decreasing at a rate of \$1.66 per player per week **17.** Monthly sales will drop at a rate of 26 T-shirts per month. **19.** Raise the price by 3¢ per week. **21.** The daily operating budget is dropping at a rate of \$2.40 per year.
23. The price is decreasing at a rate of approximately 31¢ per pound per month. **25.** $2300/\sqrt{4100} \approx 36$ miles/hour.
27. 10.7 ft/sec **29.** The y-coordinate is decreasing at a rate of 16 units per second. **31.** \$1814 per year **33.** Their prior experience must increase at a rate of approximately 0.97 years every year.
35. $\dfrac{2500}{9\pi}\left(\dfrac{3}{5000}\right)^{2/3} \approx 0.63$ m/sec
37. $\sqrt{\dfrac{1+128\pi}{4\pi}} \approx 1.6$ cm/sec **39.** 0.5137 computers per household, and increasing at a rate of 0.0230 computers per household per year. **41.** The average SAT score was 904.71 and decreasing at a rate of 0.11 per year. **43.** Decreasing by 2 percentage points per year **45.** The section is called "related rates" because the goal is to compute the rate of change of a quantity based on a knowledge of the rate of change of a related quantity.
47. Answers may vary: A rectangular solid has dimensions 2 cm × 5 cm × 10 cm, and each side is expanding at a rate of 3 cm/second. How fast is the volume increasing? **51.** Linear
53. Let x = my grades and y = your grades. If $dx/dt = 2\,dy/dt$, then $dy/dt = (1/2)\,dx/dt$.

Section 12.5

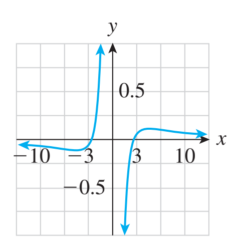
1. $E = 1.5$; the demand is going down 1.5% per 1% increase in price at that price level; revenue is maximized when $p = \$25$; weekly revenue at that price is \$12,500. **3. a.** $E = 6/7$; the demand is going down 6% per 7% increase in price at that price level; thus a price increase is in order. **b.** Revenue is maximized when $p = 100/3 \approx \$33.33$ **c.** Demand would be $(100 - 100/3)^2 = (200/3)^2 \approx 4444$ cases per week.
5. a. $E = (4p - 33)/(-2p + 33)$ **b.** 0.54; The demand for $E = mc^2$ T-shirts is going down by about 0.54% per 1% increase in the price. **c.** \$11 per shirt for a daily revenue of \$1331
7. a. $E = 1.81$. Thus, the demand is elastic at the given tuition level, showing that a decrease in tuition will result in an increase in revenue. **b.** They should charge an average of \$2250 per student, and this will result in an enrollment of about 4950 students, giving a revenue of about \$11,137,500. **9. a.** $E = 51$; the demand is going down 51% per 1% increase in price at that price level; thus a large price decrease is advised.
b. Revenue is maximized when p = ¥0.50. **c.** Demand would be $100e^{-3/4+1/2} \approx 78$ paint-by-number sets per month.
11. a. $E = -\dfrac{mp}{mp+b}$ **b.** $p = -\dfrac{b}{2m}$ **13. a.** $E = r$
b. E is independent of p. **c.** If $r = 1$, then the revenue is not affected by the price. If $r > 1$, then the revenue is always elastic, while if $r < 1$, the revenue is always inelastic. This is an unrealistic model because there should always be a price at which the revenue is a maximum. **15. a.** $q = -1500p + 6000$. **b.** \$2 per hamburger, giving a total weekly revenue of \$6000.
17. $E \approx 0.77$. At a family income level of \$20,000, the fraction of children attending a live theatrical performance is increasing by 0.77% per 1% increase in household income. **19. a.** $E \approx 0.46$. The demand for computers is increasing by 0.46% per one percent increase in household income. **b.** E decreases as income increases. **c.** Unreliable; it predicts a likelihood greater than 1 at incomes of \$123,000 and above. In a more appropriate model, one would expect the curve to level off at or below 1.
d. $E \approx 0$ **21.** $\dfrac{Y}{Q} \cdot \dfrac{dQ}{dY} = \beta$. An increase in income of x% will result in an increase in demand of βx%. (Note that we do *not* take the negative here, because we expect an increase in income to produce an *increase* in demand.) **23. a.** $q = 1000e^{-0.30p}$
b. At $p = \$3$, $E = 0.9$; at $p = \$4$, $E = 1.2$; at $p = \$5$, $E = 1.5$ **c.** $p = \$3.33$ **d.** $p = \$5.36$. Selling at a lower price would increase demand, but you cannot sell more than 200 pounds anyway. You should charge as much as you can and still be able to sell all 200 pounds. **25.** The price is lowered.
27. Start with $R = pq$, and differentiate with respect to p to obtain $\dfrac{dR}{dp} = q + p\dfrac{dq}{dp}$. For a stationary point, $dR/dp = 0$, and so $q + p\dfrac{dq}{dp} = 0$. Rearranging this result gives $p\dfrac{dq}{dp} = -q$, and hence $-\dfrac{dq}{dp} \cdot \dfrac{p}{q} = 1$, or $E = 1$, showing that stationary points of R correspond to points of unit elasticity. **29.** The distinction is best illustrated by an example. Suppose that q is measured in weekly sales and p is the unit price in dollars. Then the quantity $-dq/dp$ measures the drop in weekly sales per \$1 increase in price. The elasticity of demand E, on the other hand, measures the *percentage* drop in sales per *one percent* increase in price. Thus, $-dq/dp$ measures absolute change, while E measures fractional, or percentage, change.

Chapter 12 Review

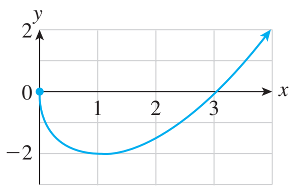
1. Relative max: $(-1, 5)$, Absolute min: $(-2, -3)$ and $(1, -3)$
3. Absolute max: $(-1, 5)$, Absolute min: $(1, -3)$ **5.** Absolute min: $(1, 0)$ **7.** Absolute min: $(-2, -1/4)$ **9.** Relative max at $x = 1$, point of inflection at $x = -1$ **11.** Relative max at $x = -2$, relative min at $x = 1$, point of inflection at $x = -1$ **13.** One point of inflection, at $x = 0$ **15. a.** $a = 4/t^4 - 2/t^3$ m/sec^2 **b.** 2 m/sec^2
17. Relative max: $(-2, 16)$; absolute min: $(2, -16)$; point of inflection: $(0, 0)$; no horizontal or vertical asymptotes



19. Relative min: $(-3, -2/9)$; relative max: $(3, 2/9)$; inflection: $(-3\sqrt{2}, -5\sqrt{2}/36)$, $(3\sqrt{2}, 5\sqrt{2}/36)$; vertical asymptote: $x = 0$; horizontal asymptote: $y = 0$



21. Relative max at (0,0), absolute min at $(1, -2)$, no asymptotes



23. \$22.14 per book **25.** \$24 per copy **27.** For maximum revenue, the company should charge \$22.14 per copy. At this price, the cost is decreasing at a linear rate with increasing price, while the revenue is not decreasing (its derivative is zero). Thus, the profit is increasing with increasing price, suggesting that the maximum profit will occur at a higher price.

29. $E = \dfrac{2p^2 - 33p}{-p^2 + 33p + 9}$ **31.** \$22.14 per book **33.** $E = 6$; The demand is dropping at a rate of 6% per 1% increase in the price. **35.** Week 5 **37.** 10,500; If weekly sales continue as predicted by the model, they will level off at around 10,500 books per week in the long-term. **39. a–d.** $10/\sqrt{2}$ ft/sec

Chapter 13

Section 13.1

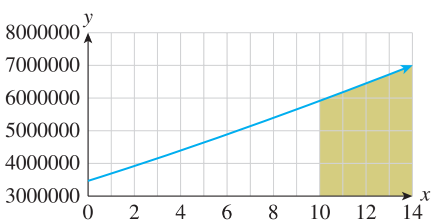
1. $x^6/6 + C$ **3.** $6x + C$ **5.** $x^2/2 + C$ **7.** $x^3/3 - x^2/2 + C$ **9.** $x + x^2/2 + C$ **11.** $-x^{-4}/4 + C$ **13.** $x^{3.3}/3.3 - x^{-0.3}/0.3 + C$ **15.** $u^3/3 - \ln|u| + C$ **17.** $\dfrac{2x^{3/2}}{3} + C$ **19.** $3x^5/5 + 2x^{-1} - x^{-4}/4 + 4x + C$ **21.** $2\ln|u| + u^2/8 + C$ **23.** $\ln|x| - \dfrac{2}{x} + \dfrac{1}{2x^2} + C$ **25.** $3x^{1.1}/1.1 - x^{5.3}/5.3 - 4.1x + C$ **27.** $\dfrac{x^{0.9}}{0.3} + \dfrac{40}{x^{0.1}} + C$ **29.** $2.55t^2 - 1.2\ln|t| - \dfrac{15}{t^{0.2}} + C$ **31.** $2e^x + 5\ln|x| + x/4 + C$ **33.** $12.2x^{0.5} + x^{1.5}/9 - e^x + C$ **35.** $\dfrac{2^x}{\ln 2} - \dfrac{3^x}{\ln 3} + C$ **37.** $\dfrac{100(1.1^x)}{\ln(1.1)} + C$ **39.** $-1/x - 1/x^2 + C$ **41.** $f(x) = x^2/2 + 1$ **43.** $f(x) = e^x - x - 1$ **45.** $C(x) = 5x - x^2/20{,}000 + 20{,}000$ **47.** $C(x) = 5x + x^2 + \ln x + 994$ **49. a.** $s = t^3/3 + t + C$ **b.** $C = 1$; $s = t^3/3 + t + 1$ **51.** 320 ft/s downwards **53. a.** $v(t) = -32t + 16$ **b.** $s(t) = -16t^2 + 16t + 185$; zenith at $t = 0.5$ sec $s = 189$ feet, 4 feet above the top of the tower. **57.** $(1280)^{1/2} \approx 35.78$ ft/sec **59. a.** 80 ft/sec **b.** 60 ft/sec **c.** 1.25 seconds **61.** $\sqrt{2} \approx 1.414$ times as fast **63.** $I(t) = 30{,}000 + 1000t$; $I(13) = \$43{,}000$ **65. a.** $H'(t) = 3.5t + 65$ billion dollars per year **b.** $H(t) = 1.75t^2 + 65t + 700$ billion dollars **67.** $S(t) \approx \dfrac{17}{3}t^3 + 50t^2 + 2300t - 7503$ million gallons. Approximately 43,000 million gallons. **69.** They differ by a constant, $G(x) - F(x) =$ Constant **71.** Antiderivative, marginal **73.** $\int f(x)\,dx$ represents the total cost of manufacturing x items. The units of $\int f(x)\,dx$ are the product of the units of $f(x)$ and the units of x. **75.** $\int(f(x) + g(x))\,dx$ is, by definition, an antiderivative of $f(x) + g(x)$. Let $F(x)$ be an antiderivative of $f(x)$ and let $G(x)$ be an antiderivative of $g(x)$. Then, because the derivative of $F(x) + G(x)$ is $f(x) + g(x)$ (by the rule for sums of derivatives), this means that $F(x) + G(x)$ is an antiderivative of $f(x) + g(x)$. In symbols, $\int(f(x) + g(x))\,dx = F(x) + G(x) + C = \int f(x)\,dx + \int g(x)\,dx$, the sum of the indefinite integrals. **77.** $\int x \cdot 1\,dx = \int x\,dx = x^2/2 + C$, whereas $\int x\,dx \cdot \int 1\,dx = (x^2/2 + D)\cdot(x + E)$, which is not the same as $x^2/2 + C$, no matter what values we choose for the constants C, D and E. **79.** If you take the *derivative* of the *indefinite integral* of $f(x)$, you obtain $f(x)$ back. On the other hand, if you take the *indefinite integral* of the *derivative* of $f(x)$, you obtain $f(x) + C$.

Section 13.2

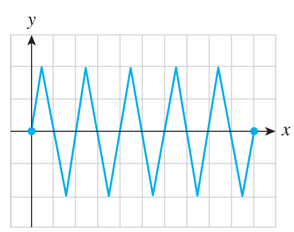
1. $(3x - 5)^4/12 + C$ **3.** $(3x - 5)^4/12 + C$ **5.** $-e^{-x} + C$ **7.** $-e^{-x} + C$ **9.** $\dfrac{1}{2}e^{(x+1)^2} + C$ **11.** $(3x + 1)^6/18 + C$ **13.** $(-2x + 2)^{-1}/2 + C$ **15.** $1.6(3x - 4)^{3/2} + C$ **17.** $2e^{(0.6x+2)} + C$ **19.** $(3x^2 + 3)^4/24 + C$ **21.** $(x^2 + 1)^{2.3}/4.6 + C$ **23.** $x + 3e^{3.1x-2} + C$ **25.** $2(3x^2 - 1)^{3/2}/9 + C$ **27.** $-(1/2)e^{-x^2+1} + C$ **29.** $-(1/2)e^{-(x^2+2x)} + C$ **31.** $(x^2 + x + 1)^{-2}/2 + C$ **33.** $(2x^3 + x^6 - 5)^{1/2}/3 + C$ **35.** $(x - 2)^7/7 + (x - 2)^6/3 + C$ **37.** $4[(x + 1)^{5/2}/5 - (x + 1)^{3/2}/3] + C$ **39.** $20\ln\left|1 - e^{-0.05x}\right| + C$ **41.** $3e^{-1/x} + C$ **43.** $(e^x - e^{-x})/2 + C$ **45.** $\ln(e^x + e^{-x}) + C$ **47.** $(e^{2x^2-2x} + e^{x^2})/2 + C$ **53.** $-e^{-x} + C$ **55.** $(1/2)e^{2x-1} + C$ **57.** $(2x + 4)^3/6 + C$ **59.** $(1/5)\ln|5x - 1| + C$ **61.** $(1.5x)^4/6 + C$ **63.** $\dfrac{1.5^{3x}}{3\ln(1.5)} + C$ **65.** $\dfrac{2^{3x+4} - 2^{-3x+4}}{3\ln 2} + C$ **67.** $f(x) = (x^2 + 1)^4/8 - 1/8$ **69.** $f(x) = (1/2)e^{x^2-1}$ **71.** $C(x) = 5x - 1/(x + 1) + 995.5$ **73. a.** $N(t) = 35\ln(5 + e^{0.2t}) - 63$ **b.** 80,000 articles **75. a.** $s = (t^2 + 1)^5/10 + t^2/2 + C$ **b.** $C = 9/10$; $s = (t^2 + 1)^5/10 + t^2/2 + 9/10$ **77.** $S(t) = \dfrac{17}{3}(t - 1990)^3 + 50(t - 1990)^2 + 2300(t - 1900) - 7503$ million gallons. $S(2003) \approx 43{,}000$ million gallons. **79.** None; the substitution $u = x$ simply replaces the letter x throughout by the letter u, and thus does not change the integral at all. For instance, the integral $\int x(3x^2 + 1)\,dx$ becomes $\int u(3u^2 + 1)\,du$ if we substitute $u = x$. **81.** The purpose of substitution is to introduce a new variable that is defined in terms of the variable of integration. One cannot say $u = u^2 + 1$, because u is not a new variable. Instead, define $w = u^2 + 1$ (or any other letter different from u). **83.** The integral $\int x(x^2 + 1)\,dx$ can be solved by the substitution $u = x^2 + 1$, because it leads to $\dfrac{1}{2}\displaystyle\int u\,du = \dfrac{1}{4}u^2 + C = \dfrac{(x^2 + 1)^2}{4} + C$.

Section 13.3

1. 30 **3.** 22 **5.** −2 **7.** 0 **9.** 4 **11.** 6 **13.** 0.7456 **15.** 2.3129 **17.** 2.5048 **19.** 1 **21.** 1/2 **23.** 1/4 **25.** 2 **27.** 0 **29.** 6 **31.** 0 **33.** 0.5 **35.** 3.3045, 3.1604, 3.1436 **37.** 0.0275, 0.0258, 0.0256 **39.** \$99.95 **41.** 19 billion gallons **43.** \$22.5 billion **45. a.** Left sum: about 46,000 articles, Right sum: about 55,000 articles **b.** 50.5; A total of about 50,500 articles in *Physics Review* were written by researchers in Europe in the 16-year period beginning 1983. **47.** 54,000 students **49.** −\$1 billion. **51.** 8160; This represents the total number of wiretaps authorized by U.S. courts from 1998 through 2003. **53.** 91.2 ft **55.** $\int_{-10}^{10} R(t)\,dt \approx \$23{,}000$. The median household income rose a total of approximately \$23,000 from 1980 to 2000. **57.** Yes. The Riemann sum gives an estimated area of 420 square feet. **59. a.** The area represents the total amount earned by households in the period 2000 through 2003, in millions of dollars. **b.** $\int_{10}^{14} A(t)\,dt \approx 26{,}000{,}000$. The total amount earned by households from 2000 through 2003 was approximately \$26 trillion.



61. a. 99.4% **b.** 0 (to at least 15 decimal places) **63.** Stays the same. **65.** Increases. **67.** The area under the curve and above the x-axis equals the area above the curve and below the x-axis. **69.** Answers will vary. One example: Let $r(t)$ be the rate of change of net income at time t. If $r(t)$ is negative, then the net income is decreasing, so the change in net income, represented by the definite integral of $r(t)$, is negative.
71. Answers may vary:



73. The total cost is $c(1) + c(2) + \cdots + c(60)$, which is represented by the Riemann sum approximation of $\int_1^{61} c(t)\,dt$ with $n = 60$.
75. $[f(x_1) + f(x_2) + \cdots + f(x_n)]\Delta x = \sum_{k=1}^{n} f(x_k)\Delta x$
77. There is no simple way of knowing for certain how accurate your answer is, but here is a rule of thumb: If increasing n does not change the value of the answer when rounded to three decimal places, then the answer can be taken to be accurate to three decimal places.

Section 13.4

1. 14/3 **3.** 5 **5.** 0 **7.** 40/3 **9.** −0.9045 **11.** $2(e-1)$ **13.** 2/3 **15.** $1/\ln 2$ **17.** $4^6 - 1 = 4095$ **19.** $(e^1 - e^{-3})/2$ **21.** $3/(2\ln 2)$ **23.** $50(e^{-1} - e^{-2})$ **25.** $e^{2.1} - e^{-0.1}$ **27.** 0 **29.** $(5/2)(e^3 - e^2)$ **31.** $(1/3)[\ln 26 - \ln 7]$ **33.** $\dfrac{0.1}{2.2\ln(1.1)}$
35. $e - e^{1/2}$ **37.** $2 - \ln 3$ **39.** −4/21 **41.** $3^{5/2}/10 - 3^{3/2}/6 + 1/15$ **43.** 1/2 **45.** 16/3 **47.** 56/3 **49.** 1/2 **51.** \$783 **53.** 296 miles **55.** 20 billion gallons **57.** \$23,000 **59.** 68 milliliters **61.** 907 T-shirts **63.** 9 gallons **67. c.** 2,100,000 iPods **69. a.** 8200 wiretaps **b.** The actual number of wiretaps is 8160, which agrees with the answer in part (a) to two significant digits. Therefore, the integral in part (a) does give an accurate estimate. **73.** They are related by the Fundamental Theorem of Calculus, which states (summarized briefly) that the definite integral of a suitable function can be calculated by evaluating the indefinite integral at the two endpoints and subtracting. **75.** The total sales from time a to time b are obtained from the marginal sales by taking its *definite integral* from a to b. **77.** An example is $v(t) = t - 5$. **79.** An example is $f(x) = e^{-x}$. **81.** By the FTC, $\int_a^x f(t)\,dt = G(x) - G(a)$ where G is an antiderivative of f. Hence, $F(x) = G(x) - G(a)$. Taking derivatives of both sides, $F'(x) = G'(x) + 0 = f(x)$, as required. The result gives us a formula, in terms of area, for an antiderivative of any continuous function.

Chapter 13 Review

1. $\dfrac{x^3}{3} - 5x^2 + 2x + C$ **3.** $4x^3/15 + 4/(5x) + C$
5. $-e^{-2x+11}/2 + C$ **7.** $\dfrac{1}{22}(x^2+4)^{11} + C$ **9.** $-\dfrac{5}{2}e^{-2x} + C$
11. $(x+2) + \ln|x+2| + C$ or $x + \ln|x+2| + C$
13. 1 **15.** −4 **17.** $5/12 \approx 0.4167$
19. 0.7778, 0.7500, 0.7471 **21.** 0 **23.** 1/4
25. $2 + e - e^{-1}$ **27.** 52/9
29. $[\ln 5 - \ln 2]/8 = \ln(2.5)/8$ **31.** 32/3 **33.** $(1 - e^{-25})/2$
35. a. $100{,}000 - 10p^2$; **b.** \$100 **37.** 25,000 copies
39. 39,200 hits **41.** About 86,000 books

Chapter 14

Section 14.1

1. $2e^x(x-1) + C$ **3.** $-e^{-x}(2+3x) + C$
5. $e^{2x}(2x^2 - 2x - 1)/4 + C$
7. $-e^{-2x+4}(2x^2 + 2x + 3)/4 + C$
9. $2^x[(2-x)/\ln 2 + 1/(\ln 2)^2] + C$
11. $-3^{-x}[(x^2-1)/\ln 3 + 2x/(\ln 3)^2 + 2/(\ln 3)^3] + C$
13. $-e^{-x}(x^2 + x + 1) + C$
15. $\dfrac{1}{7^x}(x+2)^7 - \dfrac{1}{56}(x+2)^8 + C$
17. $-\dfrac{x}{2(x-2)^2} - \dfrac{1}{2(x-2)} + C$
19. $(x^4\ln x)/4 - x^4/16 + C$
21. $(t^3/3 + t)\ln(2t) - t^3/9 - t + C$

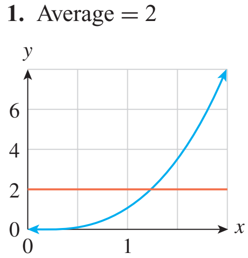
23. $(3/4)t^{4/3}(\ln t - 3/4) + C$ **25.** $x \log_3 x - x/\ln 3 + C$
27. $e^{2x}(x/2 - 1/4) - 4e^{3x}/3 + C$
29. $e^x(x^2 - 2x + 2) - e^{x^2}/2 + C$ **31.** e **33.** 38229/286
35. $(7/2)\ln 2 - 3/4$ **37.** 1/4 **39.** $1 - 11e^{-10}$
41. $4\ln 2 - 7/4$ **43.** $28{,}800{,}000(1 - 2e^{-1})$ ft.
45. $5001 + 10x - 1/(x+1) - [\ln(x+1)]/(x+1)$
47. \$33,598 **49.** \$170,000 million **51.** Answers will vary. Examples are xe^{x^2} and $e^{x^2} = 1 \cdot e^{x^2}$ **53.** $n + 1$ times

Section 14.2

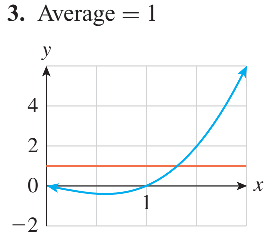
1. 8/3 **3.** 4 **5.** 1 **7.** $e - 3/2$ **9.** 2/3 **11.** 3/10 **13.** 1/20 **15.** 4/15 **17.** $2\ln 2 - 1$ **19.** $8\ln 4 + 2e - 16$ **21.** 0.9138 **23.** 0.3222 **25.** 112.5. This represents your total profit for the week, \$112.50. **27. a.** The area represents the accumulated U.S. trade deficit with China (total excess value of imports over exports) for the 8-year period 1996–2004. **b.** 640. The U.S. accumulated a \$640 billion trade deficit with China over the period 1996–2004. **29. a.** \$3600 billion. **b.** This is the area of the region between the graphs of $P(t)$ and $I(t)$ for $10 \le t \le 20$. **31.** The area between the export and import curves represents Canada's accumulated trade surplus (that is, the total excess of exports over imports) from January, 1997 to January, 2001. **33.** (A) **35.** The claim is wrong because the area under a curve can only represent income if the curve is a graph of income *per unit time*. The value of a stock price is not income per unit time—the income can only be realized when the stock is sold, and it amounts to the current market price. The total net income (per share) from the given investment would be the stock price on the date of sale minus the purchase price of \$40.

Section 14.3

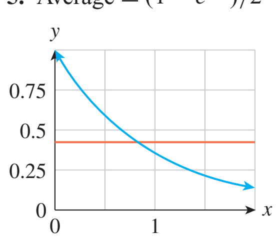
1. Average = 2



3. Average = 1



5. Average = $(1 - e^{-2})/2$



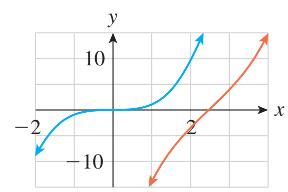
7.

x	0	1	2	3	4	5	6	7
$r(x)$	3	5	10	3	2	5	6	7
$\bar{r}(x)$			6	6	5	10/3	13/3	6

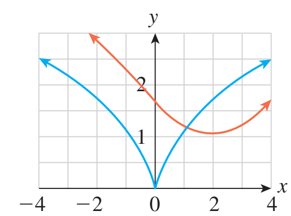
9.

x	0	1	2	3	4	5	6	7
$r(x)$	1	2	6	7	11	15	10	2
$\bar{r}(x)$			3	5	8	11	12	9

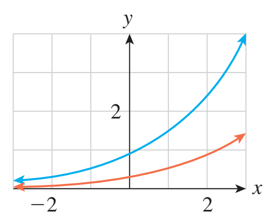
11. Moving average: $\bar{f}(x) = x^3 - (15/2)x^2 + 25x - 125/4$



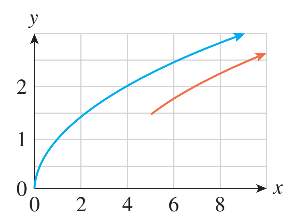
13. Moving average: $\bar{f}(x) = (3/25)[x^{5/3} - (x-5)^{5/3}]$



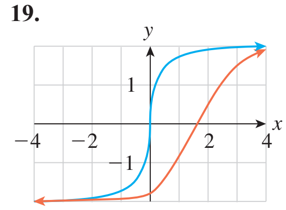
15. $\bar{f}(x) = \frac{2}{5}(e^{0.5x} - e^{0.5(x-5)})$



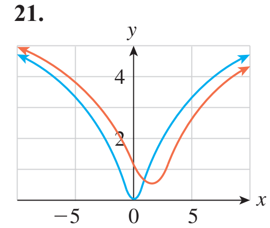
17. $\bar{f}(x) = \frac{2}{15}(x^{3/2} - (x-5)^{3/2})$



19.



21.



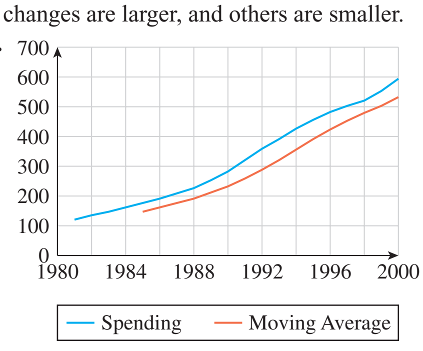
23. 127 million people **25.** \$1.7345 million **27.** \$10,410.88 **29.** \$1500

31.

Year t	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Employment (millions)	117	120	123	126	129	132	132	130	130	131
Moving average (millions)				122	125	128	130	131	131	131

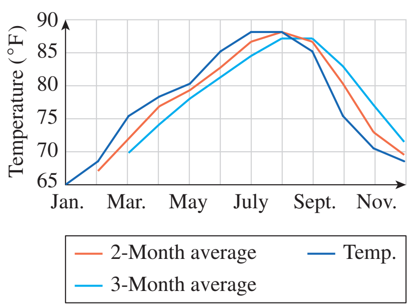
Some changes are larger, and others are smaller.

33. a.



b. \$31 billion per year; Public spending on health care in the U.S. was increasing at a rate of approximately \$31 billion per year during the given period.

35. a. 4300 million gallons per year
b. $\frac{1}{2}\left[\frac{17}{3}[t^3-(t-2)^3]+50[t^2-(t-2)^2]+4600\right]$
c. Quadratic **37. a.** $s = 14.4t + 240$
b. $\bar{s}(t) = 14.4t + 211.2$ **c.** The slope of the moving average is the same as the slope of the original function.
39. $\bar{f}(x) = mx + b - \frac{ma}{2}$
41. a.



b. The 24-month moving average is constant and equal to the year-long average of approximately 77°. **c.** A quadratic model could not be used to predict temperatures beyond the given 12-month period, since temperature patterns are periodic, whereas parabolas are not. **43.** The moving average "blurs" the effects of short-term oscillations in the price, and shows the longer-term trend of the stock price. **45.** The area above the x-axis equals the area below the x-axis. Example: $y = x$ on $[-1, 1]$ **47.** This need not be the case; for instance, the function $f(x) = x^2$ on [0, 1] has average value 1/3, whereas the value midway between the maximum and minimum is 1/2. **49.** (C). A shorter term moving average most closely approximates the original function, since it averages the function over a shorter period, and continuous functions change by only a small amount over a small period.

Section 14.4

1. \$6.25 **3.** \$512 **5.** \$119.53 **7.** \$900 **9.** \$416.67 **11.** \$326.27 **13.** \$25 **15.** \$0.50 **17.** \$386.29 **19.** \$225 **21.** \$25.50 **23.** \$12,684.63 **25.** $TV = \$300{,}000$, $FV = \$434{,}465.45$ **27.** $TV = \$350{,}000$, $FV = \$498{,}496.61$
29. $TV = \$389{,}232.76$, $FV = \$547{,}547.16$
31. $TV = \$100{,}000$, $PV = \$82{,}419.99$
33. $TV = \$112{,}500$, $PV = \$92{,}037.48$
35. $TV = \$107{,}889.50$, $PV = \$88{,}479.69$ **37.** $\bar{p} = \$5000$, $\bar{q} = 10{,}000$, $CS = \$25$ million, $PS = \$100$ million. The total social gain is \$125 million. **39.** $CS = \frac{1}{2m}(b - m\bar{p})^2$
41. €140 billion **43.** \$940 billion **45.** €160 billion
47. \$850 billion **49.** \$1,943,162.44 **51.** \$3,086,245.73
53. \$58,961.74 **55.** \$1,792,723.35 **57.** Total **59.** She is correct, provided there is a positive rate of return, in which case the future value (which includes interest) is greater than the total value (which does not). **61.** $PV < TV < FV$

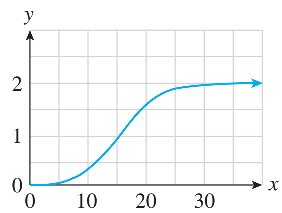
Section 14.5

1. Diverges **3.** Converges to $2e$ **5.** Converges to e^2
7. Converges to 1/2 **9.** Converges to 1/108 **11.** Converges to $3 \times 5^{2/3}$ **13.** Diverges **15.** Diverges **17.** Converges to $\frac{5}{4}(3^{4/5} - 1)$. **19.** Diverges **21.** Converges to 0
23. Diverges **25.** Diverges **27.** \$870 million **29.** 7100 billion cigarettes **31.** No; You will not sell more than 2000 of them. **33.** The integral diverges, and so the number of graduates each year will rise without bound. **35. a.** $R(t) = 350e^{-0.1t}(39t + 68)$ million dollars/yr; **b.** \$1,603,000 million
37. \$27,000 billion **39.** $\int_0^{+\infty} N(t)\,dt$ diverges, indicating that there is no bound to the expected future total online sales of books. $\int_{-\infty}^{0} N(t)\,dt$ converges to approximately 1.889, indicating that total online sales of books prior to 1997 amounted to approximately 1.889 million books **41.** 1 **43.** 0.1587
45. \$70,833 **47. a.** 2.468 meteors on average **b.** The integral diverges. We can interpret this as saying that the number of impacts by meteors smaller than 1 megaton is very large. (This makes sense because, for example, this number includes meteors no larger than a grain of dust.) **49. a.** $\Gamma(1) = 1$; $\Gamma(2) = 1$
51. The integral does not converge, so the number given by the FTC is meaningless. **53.** Yes; the integrals converge to 0, and the FTC also gives 0. **55.** In all cases, you need to rewrite the improper integral as a limit and use technology to evaluate the integral of which you are taking the limit. Evaluate for several values of the endpoint approaching the limit. In the case of an integral in which one of the limits of integration is infinite, you may have to instruct the calculator or computer to use more subdivisions as you approach $+\infty$.

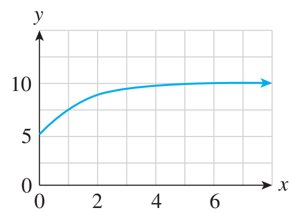
Section 14.6

1. $y = \frac{x^3}{3} + \frac{2x^{3/2}}{3} + C$ **3.** $\frac{y^2}{2} = \frac{x^2}{2} + C$ **5.** $y = Ae^{x^2/2}$
7. $y = -\frac{2}{(x+1)^2 + C}$ **9.** $y = \pm\sqrt{(\ln x)^2 + C}$
11. $y = \frac{x^4}{4} - x^2 + 1$ **13.** $y = (x^3 + 8)^{1/3}$ **15.** $y = 2x$
17. $y = e^{x^2/2} - 1$ **19.** $y = -\frac{2}{\ln(x^2+1) + 2}$
21. With $s(t)$ = monthly sales after t months, $\frac{ds}{dt} = -0.05s$; $s = 1000$ when $t = 0$. Solution: $s = 1000e^{-0.05t}$ quarts per month. **23.** $H(t) = 75 + 115e^{-0.04274t}$ degrees Fahrenheit after t minutes. **25.** With $S(t)$ = total sales after t months, $\frac{dS}{dt} = 0.1(100{,}000 - S)$; $S(0) = 0$.
Solution: $S = 100{,}000(1 - e^{-0.1t})$ monitors after t months.
27. a. $\frac{dp}{dt} = k(D(p) - S(p)) = k(20{,}000 - 1{,}000p)$
b. $p = 20 - Ae^{-kt}$ **c.** $p = 20 - 10e^{-0.2231t}$ dollars after t months. **29.** $q = 0.6078e^{-0.05p}p^{1.5}$

33. $S = \dfrac{2/1999}{e^{-0.5t} + 1/1999}$
It will take about 27 months to saturate the market. Graph:



35. a. $y = be^{Ae^{-at}}$, A = constant **b.** $y = 10e^{-0.69315e^{-t}}$ Graph:



37. A general solution gives all possible solutions to the equation, using at least one arbitrary constant. A particular solution is one specific function that satisfies the equation. We obtain a particular solution by substituting specific values for any arbitrary constants in the general solution. **39.** Example: $y'' = x$ has general solution $y = \frac{1}{6}x^3 + Cx + D$ (integrate twice).
41. $y' = -4e^{-x} + 3$

Chapter 14 Review

1. $(x^2 - 2x + 4)e^x + C$ **3.** $(1/3)x^3 \ln 2x - x^3/9 + C$
5. $-e^2 - 39/e^2$ **7.** $1/4$ **9.** $3(3^{2/3} - 1)/2$ **11.** $\dfrac{3}{2 \cdot 2^{1/3}} - \dfrac{1}{2}$
13. $\dfrac{2\sqrt{2}}{3}$ **15.** -1 **17.** $e - 2$ **19.** $3x - 2$
21. $\dfrac{3}{14}[x^{7/3} - (x-2)^{7/3}]$ **23.** \$1600 **25.** \$2500
27. $y = -\dfrac{3}{x^3 + C}$ **29.** $y = \sqrt{2 \ln|x| + 1}$
31. a. \$1,062,500; **b.** $997{,}500e^{0.06t}$ **33.** Approximately \$910,000 **35.** \$51 million

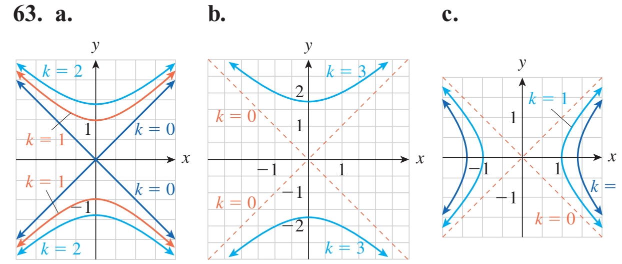
Chapter 15

Section 15.1

1. a. 1 **b.** 1 **c.** 2 **d.** $a^2 - a + 5$ **e.** $y^2 + x^2 - y + 1$
f. $(x+h)^2 + (y+k)^2 - (x+h) + 1$ **3. a.** 0 **b.** 0.2
c. -0.1 **d.** $0.18a + 0.2$ **e.** $0.1x + 0.2y - 0.01xy$
f. $0.2(x+h) + 0.1(y+k) - 0.01(x+h)(y+k)$
5. a. 1 **b.** e **c.** e **d.** e^{x+y+z} **e.** $e^{x+h+y+k+z+l}$
7. a. Does not exist **b.** 0 **c.** 0 **d.** $xyz/(x^2 + y^2 + z^2)$
e. $(x+h)(y+k)(z+l)/[(x+h)^2 + (y+k)^2 + (z+l)^2]$
9. a. Increases; 2.3 **b.** Decreases; 1.4 **c.** Decreases; 1 unit increase in z **11.** Neither **13.** Linear **15.** Linear
17. Interaction **19. a.** 107 **b.** -14 **c.** -113
21.

$y \downarrow$ \ $x \rightarrow$	**10**	**20**	**30**	**40**
10	52	107	162	217
20	94	194	294	394
30	136	281	426	571
40	178	368	558	748

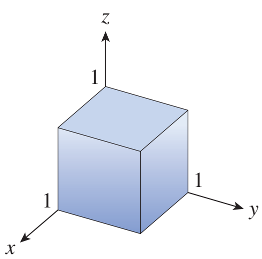
25. 18, 4, 0.0965, 47,040 **27.** 6.9078, 1.5193, 5.4366, 0
29. Let z = annual sales of Z (in millions of dollars), x = annual sales of X, and y = annual sales of Y. The model is $z = -2.1x + 0.4y + 16.2$ **31.** $\sqrt{2}$ **33.** $\sqrt{a^2 + b^2}$
35. 1/2 **37.** Circle with center $(2, -1)$ and radius 3
39. The marginal cost of cars is \$6000 per car. The marginal cost of trucks is \$4000 per truck.
41. $C(x, y) = 10 + 0.03x + 0.04y$ where C is the cost in dollars, x = # video clips sold per month, y = # audio clips sold per month **43. a.** 28% **b.** 21% **c.** Percentage points per year
45. a. 29% **b.** 7% **c.** 0.4-point drop **47. a.** \$9980
b. $R(z) = 9850 + 0.04z$ **49.** $s(c, t) = 1.2c - 2t$
51. $U(11, 10) - U(10, 10) \approx 5.75$. This means that, if your company now has 10 copies of Macro Publish and 10 copies of Turbo Publish, then the purchase of one additional copy of Macro Publish will result in a productivity increase of approximately 5.75 pages per day. **53. a.** Answers will vary. $(a, b, c) = (3, 1/4, 1/\pi)$; $(a, b, c) = (1/\pi, 3, 1/4)$.
b. $a = \left(\dfrac{3}{4\pi}\right)^{1/3}$. The resulting ellipsoid is a sphere with radius a.
55. 7,000,000 **57. a.** $100 = K(1{,}000)^a(1{,}000{,}000)^{1-a}$; $10 = K(1{,}000)^a(10{,}000)^{1-a}$ **b.** $\log K - 3a = -4$; $\log K - a = -3$ **c.** $a = 0.5$, $K \approx 0.003162$
d. $P = 71$ pianos (to the nearest piano) **59. a.** 4×10^{-3} gram per square meter **b.** The total weight of sulfates in the Earth's atmosphere **61. a.** The value of N would be doubled.
b. $N(R, f_p, n_e, f_l, f_i, L) = Rf_pn_ef_lf_iL$, where here L is the average lifetime of an intelligent civilization **c.** Take the logarithm of both sides, since this would yield the linear function $\ln(N) = \ln(R) + \ln(f_p) + \ln(n_e) + \ln(f_l) + \ln(f_i) + \ln(f_c) + \ln(L)$.
63. a. **b.** **c.**



65. They are reciprocals of each other. **67.** For example, $f(x, y) = x^2 + y^2$. **69.** For example, $f(x, y, z) = xyz$
71. For example, take $f(x, y) = x + y$. Then setting $y = 3$ gives $f(x, 3) = x + 3$. This can be viewed as a function of the single variable x. Choosing other values for y gives other functions of x. **73.** If $f = ax + by + c$, then fixing $y = k$ gives $f = ax + (bk + c)$, a linear function with slope a and intercept $bk + c$. The slope is independent of the choice of $y = k$.
75. CDs cost more than cassettes.

Section 15.2

1.



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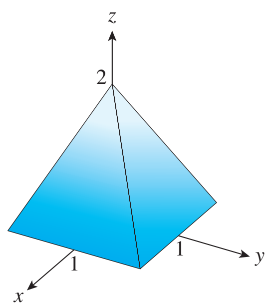

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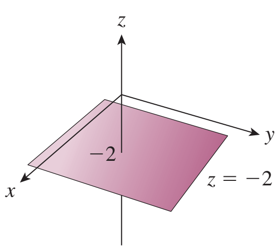

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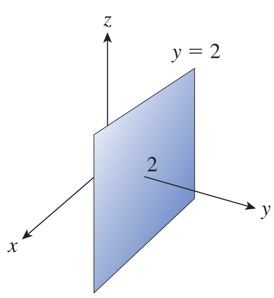

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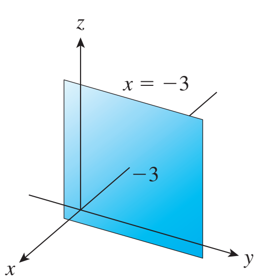
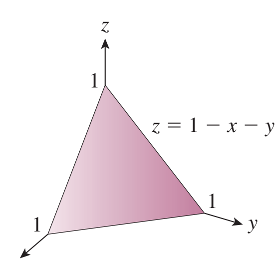


11. (H) **13.** (B) **15.** (F) **17.** (C)

19.



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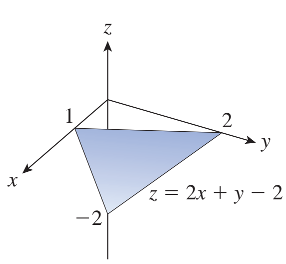

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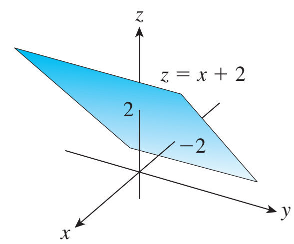

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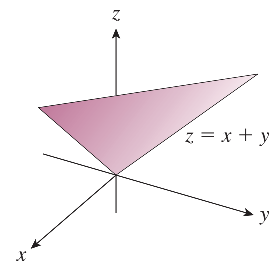

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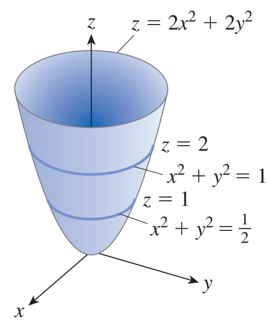

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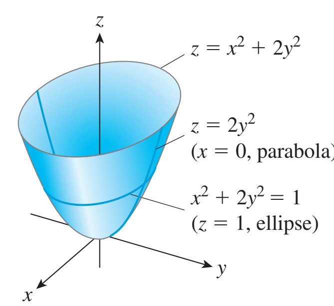

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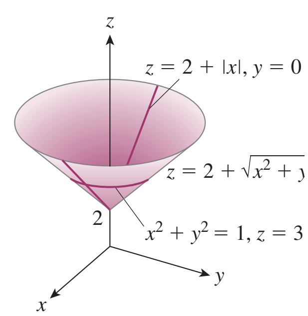

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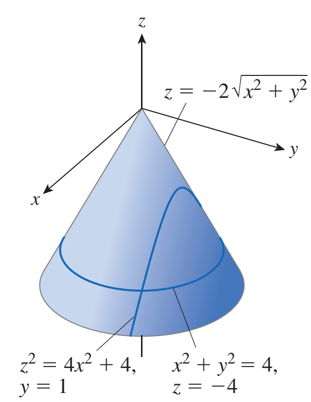

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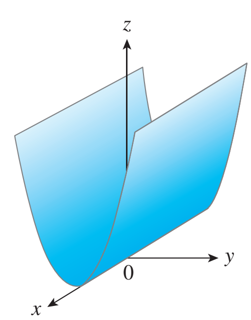

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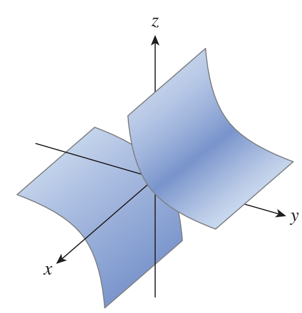
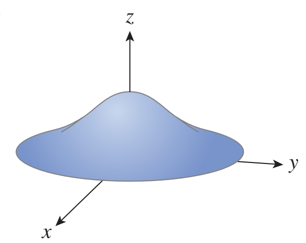
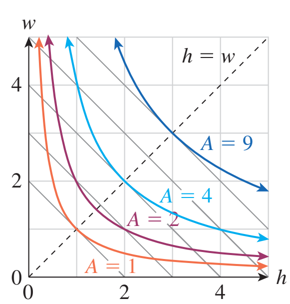


39.

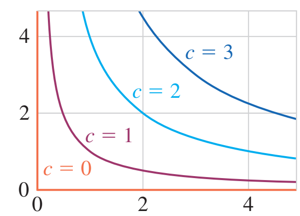


41. a. The graph is a plane with x-intercept -40, y-intercept -60, and z-intercept 240,000. **b.** The slice $x = 10$ is the straight line with equation $z = 300{,}000 + 4000y$. It describes the cost function for the manufacture of trucks if car production is held fixed at 10 cars per week. **c.** The level curve $z = 480{,}000$ is the straight line $6000x + 4000y = 240{,}000$. It describes the number of cars and trucks you can manufacture to maintain weekly costs at \$480,000. **43.** The graph is a plane with x_1-intercept 0.3, x_2-intercept 33, and x_3-intercept 0.66. The slices by $x_1 =$ constant are straight lines that are parallel to each other. Thus, the rate of change of General Motors' share as a function of Ford's share does not depend on Chrysler's share. Specifically, GM's share decreases by 0.02 percentage points per 1 percentage-point increase in Ford's market share, regardless of Chrysler's share. **45. a.** The slices $x =$ constant and $y =$ constant are straight lines. **b.** No. Even though the slices $x =$ constant and $y =$ constant are straight lines, the level curves are not, and so the surface is not a plane. **c.** The slice $x = 10$ has a slope of 3800. The slice $x = 20$ has a slope of 3600. Manufacturing more cars lowers the marginal cost of manufacturing trucks. **47.** Both level curves are quarter-circles. (We see only the portion in the first quadrant because $e \geq 0$ and $k \geq 0$.) The level curve $C = 30{,}000$ represents the relationship between the number of electricians and the number of carpenters used in building a home that costs \$30,000. Similarly for the level curve $C = 40{,}000$. **49.** The following figure shows several level curves together with several lines of the form $h + w = c$.

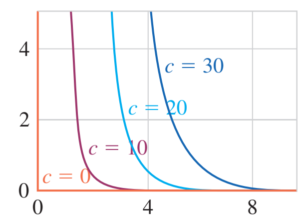


From the figure, thinking of the curves as contours on a map, we see that the largest value of A anywhere along any of the lines $h + w = c$ occurs midway along the line, when $h = w$. Thus, the largest area rectangle with a fixed perimeter occurs when $h = w$ (that is, when the rectangle is a square).

51. The level curve at $z = 3$ has the form $3 = x^{0.5}y^{0.5}$, or $y = 9/x$, and shows the relationship between the number of workers and the operating budget at a production level of 3 units.



53. The level curve at $z = 0$ consists of the nonnegative y-axis $(x = 0)$ and tells us that zero utility corresponds to zero copies of Macro Publish, regardless of the number of copies of Turbo Publish. (Zero copies of Turbo Publish does not necessarily result in zero utility, according to the formula.)



55. Plane **57.** Agree: any slice through a plane is a straight line. **59.** The graph of a function of three or more variables lives in four-dimensional (or higher) space, which makes it difficult to draw and visualize. **63.** We need one dimension for each of the variables plus one dimension for the value of the function.

Section 15.3

1. $f_x(x, y) = -40$; $f_y(x, y) = 20$; $f_x(1, -1) = -40$; $f_y(1, -1) = 20$ **3.** $f_x(x, y) = 6x + 1$; $f_y(x, y) = -3y^2$; $f_x(1, -1) = 7$; $f_y(1, -1) = -3$ **5.** $f_x(x, y) = -40 + 10y$; $f_y(x, y) = 20 + 10x$; $f_x(1, -1) = -50$; $f_y(1, -1) = 30$ **7.** $f_x(x, y) = 6xy$; $f_y(x, y) = 3x^2$; $f_x(1, -1) = -6$; $f_y(1, -1) = 3$ **9.** $f_x(x, y) = 2xy^3 - 3x^2y^2 - y$; $f_y(x, y) = 3x^2y^2 - 2x^3y - x$; $f_x(1, -1) = -4$; $f_y(1, -1) = 4$ **11.** $f_x(x, y) = 6y(2xy + 1)^2$; $f_y(x, y) = 6x(2xy + 1)^2$; $f_x(1, -1) = -6$; $f_y(1, -1) = 6$ **13.** $f_x(x, y) = e^{x+y}$; $f_y(x, y) = e^{x+y}$; $f_x(1, -1) = 1$; $f_y(1, -1) = 1$ **15.** $f_x(x, y) = 3x^{-0.4}y^{0.4}$; $f_y(x, y) = 2x^{0.6}y^{-0.6}$; $f_x(1, -1)$ undefined; $f_y(1, -1)$ undefined **17.** $f_x(x, y) = 0.2ye^{0.2xy}$; $f_y(x, y) = 0.2xe^{0.2xy}$; $f_x(1, -1) = -0.2e^{-0.2}$; $f_y(1, -1) = 0.2e^{-0.2}$ **19.** $f_{xx}(x, y) = 0$; $f_{yy}(x, y) = 0$; $f_{xy}(x, y) = f_{yx}(x, y) = 0$; $f_{xx}(1, -1) = 0$; $f_{yy}(1, -1) = 0$; $f_{xy}(1, -1) = f_{yx}(1, -1) = 0$ **21.** $f_{xx}(x, y) = 0$; $f_{yy}(x, y) = 0$; $f_{xy}(x, y) = f_{yx}(x, y) = 10$; $f_{xx}(1, -1) = 0$; $f_{yy}(1, -1) = 0$; $f_{xy}(1, -1) = f_{yx}(1, -1) = 10$ **23.** $f_{xx}(x, y) = 6y$; $f_{yy}(x, y) = 0$; $f_{xy}(x, y) = f_{yx}(x, y) = 6x$; $f_{xx}(1, -1) = -6$; $f_{yy}(1, -1) = 0$; $f_{xy}(1, -1) = f_{yx}(1, -1) = 6$ **25.** $f_{xx}(x, y) = e^{x+y}$; $f_{yy}(x, y) = e^{x+y}$; $f_{xy}(x, y) = f_{yx}(x, y) = e^{x+y}$; $f_{xx}(1, -1) = 1$; $f_{yy}(1, -1) = 1$; $f_{xy}(1, -1) = f_{yx}(1, -1) = 1$

27. $f_{xx}(x, y) = -1.2x^{-1.4}y^{0.4}$; $f_{yy}(x, y) = -1.2x^{0.6}y^{-1.6}$; $f_{xy}(x, y) = f_{yx}(x, y) = 1.2x^{-0.4}y^{-0.6}$; $f_{xx}(1, -1)$ undefined; $f_{yy}(1, -1)$ undefined; $f_{xy}(1, -1)$ & $f_{yx}(1, -1)$ undefined **29.** $f_x(x, y, z) = yz$; $f_y(x, y, z) = xz$; $f_z(x, y, z) = xy$; $f_x(0, -1, 1) = -1$; $f_y(0, -1, 1) = 0$; $f_z(0, -1, 1) = 0$ **31.** $f_x(x, y, z) = 4/(x + y + z^2)^2$; $f_y(x, y, z) = 4/(x + y + z^2)^2$; $f_z(x, y, z) = 8z/(x + y + z^2)^2$; $f_x(0, -1, 1)$ undefined; $f_y(0, -1, 1)$ undefined; $f_z(0, -1, 1)$ undefined **33.** $f_x(x, y, z) = e^{yz} + yze^{xz}$; $f_y(x, y, z) = xze^{yz} + e^{xz}$; $f_z(x, y, z) = xy(e^{yz} + e^{xz})$; $f_x(0, -1, 1) = e^{-1} - 1$; $f_y(0, -1, 1) = 1$; $f_z(0, -1, 1) = 0$ **35.** $f_x(x, y, z) = 0.1x^{-0.9}y^{0.4}z^{0.5}$; $f_y(x, y, z) = 0.4x^{0.1}y^{-0.6}z^{0.5}$; $f_z(x, y, z) = 0.5x^{0.1}y^{0.4}z^{-0.5}$; $f_x(0, -1, 1)$ undefined; $f_y(0, -1, 1)$ undefined, $f_z(0, -1, 1)$ undefined **37.** $f_x(x, y, z) = yze^{xyz}$, $f_y(x, y, z) = xze^{xyz}$, $f_z(x, y, z) = xye^{xyz}$; $f_x(0, -1, 1) = -1$; $f_y(0, -1, 1) = f_z(0, -1, 1) = 0$ **39.** $f_x(x, y, z) = 0$; $f_y(x, y, z) = -\dfrac{600z}{y^{0.7}(1 + y^{0.3})^2}$; $f_z(x, y, z) = \dfrac{2{,}000}{1 + y^{0.3}}$; $f_x(0, -1, 1)$ undefined; $f_y(0, -1, 1)$ undefined; $f_z(0, -1, 1)$ undefined **41.** $\partial C/\partial x = 6000$, the marginal cost to manufacture each car is \$6000. $\partial C/\partial y = 4000$, the marginal cost to manufacture each truck is \$4000. **43.** $\partial y/\partial t = -0.78$. The number of articles written by researchers in the U.S. was decreasing at a rate of 0.78 percentage points per year. $\partial y/\partial x = -1.02$. The number of articles written by researchers in the U.S. was decreasing at a rate of 1.02 percentage points per one percentage point increase in articles written in Europe. **45.** \$5600 per car **47. a.** $\partial M/\partial c = -3.8$, $\partial M/\partial f = 2.2$. For every 1 point increase in the percentage of Chrysler owners who remain loyal, the percentage of Mazda owners who remain loyal decreases by 3.8 points. For every 1 point increase in the percentage of Ford owners who remain loyal, the percentage of Mazda owners who remain loyal increases by 2.2 points. **b.** 16% **49. a.** \$16,500 **b.** \$28,600 **c.** \$350 per year **d.** \$570 per year **e.** Widening **51.** The marginal cost of cars is $\$6000 + 1{,}000e^{-0.01(x+y)}$ per car. The marginal cost of trucks is $\$4000 + 1{,}000e^{-0.01(x+y)}$ per truck. Both marginal costs decrease as production rises.

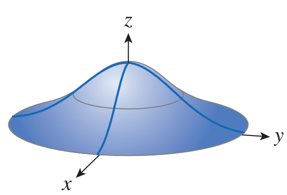
53. $\bar{C}(x, y) = \dfrac{200{,}000 + 6{,}000x + 4{,}000y - 100{,}000e^{-0.01(x+y)}}{x + y}$; $\bar{C}_x(50, 50) = -\$2.64$ per car. This means that at a production level of 50 cars and 50 trucks per week, the average cost per vehicle is decreasing by \$2.64 for each additional car manufactured. $\bar{C}_y(50, 50) = -\$22.64$ per truck. This means that at a production level of 50 cars and 50 trucks per week, the average cost per vehicle is decreasing by \$22.64 for each additional truck manufactured. **55.** No; your marginal revenue from the sale of cars is $\$15{,}000 - \dfrac{2{,}500}{\sqrt{x + y}}$ per car and $\$10{,}000 - \dfrac{2{,}500}{\sqrt{x + y}}$ per truck from the sale of trucks. These increase with increasing x and y. In other words, you will earn more revenue per vehicle with increasing sales, and so the rental company will pay more for each additional vehicle it buys.

57. $P_z(10{,}100{,}000,\ 1{,}000{,}000) \approx 0.0001010$ papers/\$ **59.** $U_x(10, 5) = 5.18$, $U_y(10, 5) = 2.09$. This means that, if 10 copies of Macro Publish and 5 copies of Turbo Publish are purchased, the company's daily productivity is increasing at a rate of 5.18 pages per day for each additional copy of Macro purchased and by 2.09 pages per day for each additional copy of Turbo purchased. **b.** $\dfrac{U_x(10, 5)}{U_y(10, 5)} \approx 2.48$ is the ratio of the usefulness of one additional copy of Macro to one of Turbo. Thus, with 10 copies of Macro and 5 copies of Turbo, the company can expect approximately 2.48 times the productivity per additional copy of Macro compared to Turbo. **61.** 6×10^9 N/sec **63. a.** $A_P(100, 0.1, 10) = 2.59$; $A_r(100, 0.1, 10) = 2{,}357.95$; $A_t(100, 0.1, 10) = 24.72$. Thus, for a \$100 investment at 10% interest, after 10 years the accumulated amount is increasing at a rate of \$2.59 per \$1 of principal, at a rate of \$2,357.95 per increase of 1 in r (note that this would correspond to an increase in the interest rate of 100%), and at a rate of \$24.72 per year. **b.** $A_P(100, 0.1, t)$ tells you the rate at which the accumulated amount in an account bearing 10% interest with a principal of \$100 is growing per \$1 increase in the principal, t years after the investment. **65. a.** $P_x = Ka\left(\dfrac{y}{x}\right)^b$ and $P_y = Kb\left(\dfrac{x}{y}\right)^a$. They are equal precisely when $\dfrac{a}{b} = \left(\dfrac{x}{y}\right)^b\left(\dfrac{x}{y}\right)^a$. Substituting $b = 1 - a$ now gives $\dfrac{a}{b} = \dfrac{x}{y}$. **b.** The given information implies that $P_x(100, 200) = P_y(100, 200)$. By part (a), this occurs precisely when $a/b = x/y = 100/200 = 1/2$. But $b = 1 - a$, so $a/(1 - a) = 1/2$, giving $a = 1/3$ and $b = 2/3$. **67.** Decreasing at 0.0075 parts of nutrient per part of water/sec **69.** f is increasing at a rate of s units per unit of x, f is increasing at a rate of t units per unit of y, and the value of f is r when $x = a$ and $y = b$ **71.** The marginal cost of building an additional orbicus; zonars per unit. **73.** Answers will vary. One example is $f(x, y) = -2x + 3y$. Others are $f(x, y) = -2x + 3y + 9$ and $f(x, y) = xy - 3x + 2y + 10$. **75. a.** b is the z-intercept of the plane. m is the slope of the intersection of the plane with the xz-plane. n is the slope of the intersection of the plane with the yz-plane. **b.** Write $z = b + rx + sy$. We are told that $\partial z/\partial x = m$, so $r = m$. Similarly, $s = n$. Thus, $z = b + mx + ny$. We are also told that the plane passes through (h, k, l). Substituting gives $l = b + mh + nk$. This gives b as $l - mh - nk$. Substituting in the equation for z therefore gives $z = l - mh - nk + mx + ny = l + m(x - h) + n(y - k)$, as required.

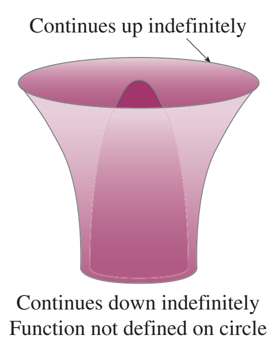
Section 15.4

1. P: relative minimum; Q: none of the above; R: relative maximum **3.** P: saddle point; Q: relative maximum; R: none of the above **5.** Relative minimum **7.** Neither **9.** Saddle point **11.** Relative minimum at (0, 0, 1) **13.** Relative maximum at (−1/2, 1/2, 3/2) **15.** Relative maximum at (0, 0, 0), saddle points at (±4, 2, −16) **17.** Relative minimum at (0, 0, 1) **19.** Relative minimum at (−2, ±2, −16), (0, 0) a critical point that is not a relative extremum **21.** Saddle point at $(0, 0, -1)$ **23.** Relative maximum at $(-1, 0, e)$ **25.** Relative minimum at $(2^{1/3}, 2^{1/3}, 3(2^{2/3}))$ **27.** Relative minimum at $(1, 1, 4)$ and $(-1, -1, 4)$ **29.** Absolute minimum at $(0, 0, 1)$ **31.** None; the relative maximum at $(0, 0, 0)$ is not absolute. (look at, say, $(10, 10)$). **33.** Minimum of $1/3$ at $(c, f) = (2/3, 2/3)$. Thus, at least $1/3$ of all Mazda owners would choose another new Mazda, and this lowest loyalty occurs when $2/3$ of Chrysler and Ford owners remain loyal to their brands. **35.** It should remove 2.5 pounds of sulfur and 1 pound of lead per day. **37.** You should charge \$580.81 for the Ultra Mini and \$808.08 for the Big Stack. **39.** $l = w = h \approx 20.67$ in, volume ≈ 8827 cubic inches. **41.** 18 in $\times$ 18 in $\times$ 36 in, volume $= 11{,}664$ cubic inches

43.



45.



47. H must be positive. **49.** No. In order for there to be a relative maximum at (a, b), *all* vertical planes through (a, b) should yield a curve with a relative maximum at (a, b). It could happen that a slice by another vertical plane through (a, b) (such as $x - a = y - b$) does not yield a curve with a relative maximum at (a, b). [An example is $f(x, y) = x^2 + y^2 - \sqrt{xy}$, at the point $(0, 0)$. Look at the slices through $x = 0$, $y = 0$ and $y = x$.] **51.** $\bar{C}_x = \dfrac{\partial}{\partial x}\left(\dfrac{C}{x+y}\right) = \dfrac{(x+y)C_x - C}{(x+y)^2}$. If this is zero, then $(x + y)C_x = C$, or $C_x = \dfrac{C}{x+y} = \bar{C}$. Similarly, if $\bar{C}_y = 0$ then $C_y = \bar{C}$. This is reasonable because if the average cost is decreasing with increasing x, then the average cost is greater than the marginal cost C_x. Similarly, if the average cost is increasing with increasing x, then the average cost is less than the marginal cost C_x. Thus, if the average cost is stationary with increasing x, then the average cost equals the marginal cost C_x. (The situation is similar for the case of increasing y.)
53. The equation of the tangent plane at the point (a, b) is $z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$. If f has a relative extremum at (a, b), then $f_x(a, b) = 0 = f_y(a, b)$. Substituting these into the equation of the tangent plane gives $z = f(a, b)$, a constant. But the graph of $z = \textit{constant}$ is a plane parallel to the xy-plane.

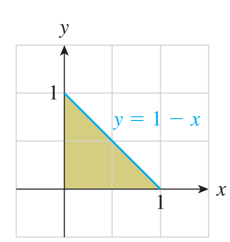
Section 15.5

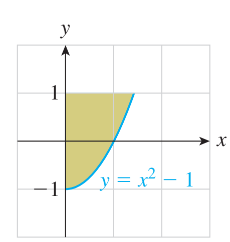
1. 1; $(0, 0, 0)$ **3.** 1.35; $(1/10, 3/10, 1/2)$ **5.** Minimum value $= 6$ at $(1, 2, 1/2)$ **7.** 200; $(20, 10)$ **9.** 16; $(2, 2)$ and $(-2, -2)$ **11.** 20; $(2, 4)$ **13.** 1; $(0, 0, 0)$ **15.** 1.35; $(1/10, 3/10, 1/2)$ **17.** Minimum value $= 6$ at $(1, 2, 1/2)$ **19.** $5 \times 10 = 50$ sq. ft. **21.** \$10 **23.** $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$, $(-1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$, $(1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$, $(-1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$ **25.** $(0, 1/2, -1/2)$ **27.** $(-5/9, 5/9, 25/9)$ **29.** $l \times w \times h = 1 \times 1 \times 2$ **31.** 18 in $\times$ 18 in $\times$ 36 in, volume $= 11{,}664$ cubic inches
33. $(2l/h)^{1/3} \times (2l/h)^{1/3} \times 2^{1/3}(h/l)^{2/3}$, where $l =$ cost of lightweight cardboard and $h =$ cost of heavy-duty cardboard per square foot **35.** $1 \times 1 \times 1/2$ **37.** Method 1: Solve $g(x, y, z) = 0$ for one of the variables and substitute in $f(x, y, z)$. Then find the maximum value of the resulting function of 2 variables. Advantage (Answers may vary): We can use the second derivative test to check whether the resulting critical points are maxima, minima, saddle points, or none of these. Disadvantage (Answers may vary): We may not be able to solve $g(x, y, z) = 0$ for one of the variables. Method 2: Use the method of Lagrange Multipliers. Advantage (Answers may vary): We do not need to solve the constraint equation for one of the variables. Disadvantage (Answers may vary): The method does not tell us whether the critical points obtained are maxima, minima, saddle points, or none of these. **39.** If the only constraint is an equality constraint, and if it is impossible to eliminate one of the variables in the objective function by substitution (solving the constraint equation for a variable or some other method). **41.** Answers may vary: Maximize $f(x, y) = 1 - x^2 - y^2$ subject to $x = y$. **43.** Yes. There may be relative extrema at points on the boundary of the domain of the function. The partial derivatives of the function need not be 0 at such points. **45.** If the solution were located in the interior of one of the line segments making up the boundary of the domain of f, then the derivative of a certain function would be 0. This function is obtained by substituting the linear equation $C(x, y) = 0$ in the linear objective function. But because the result would again be a linear function, it is either constant, or its derivative is a nonzero constant. In either event, extrema lie on the boundary of that line segment; that is, at one of the corners of the domain.

Section 15.6

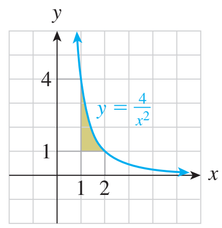
1. $-1/2$ **3.** $e^2/2 - 7/2$ **5.** $(e^3 - 1)(e^2 - 1)$ **7.** $7/6$ **9.** $(e^3 - e - e^{-1} + e^{-3})/2$ **11.** $1/2$ **13.** $(e - 1)/2$ **15.** $45/2$ **17.** $8/3$ **19.** $4/3$ **21.** 0 **23.** $2/3$ **25.** $2/3$ **27.** $2(e - 2)$ **29.** $2/3$

31. $\displaystyle\int_0^1 \int_0^{1-x} f(x, y)\,dy\,dx$ **33.** $\displaystyle\int_0^{\sqrt{2}} \int_{x^2-1}^1 f(x, y)\,dy\,dx$





35. $\int_1^4 \int_1^{2/\sqrt{y}} f(x, y)\,dx\,dy$



37. 4/3 **39.** 1/6 **41.** 162,000 gadgets **43.** Average revenue is \$312,750. **45.** Average revenue is \$17,500. **47.** 8216 **49.** 1 degree **51.** The area between the curves $y = r(x)$ and $y = s(x)$ and the vertical lines $x = a$ and $x = b$ is given by $\int_a^b \int_{r(x)}^{s(x)} dy\,dx$ assuming that $r(x) \le s(x)$ for $a \le x \le b$.

53. The first step in calculating an integral of the form $\int_a^b \int_{r(x)}^{s(x)} f(x, y)\,dy\,dx$ is to evaluate the integral $\int_{r(x)}^{s(x)} f(x, y)\,dy$, obtained by holding x constant and integrating with respect to y. **55.** Paintings per picasso per dali **57.** Left-hand side is $\int_a^b \int_c^d f(x)g(y)\,dx\,dy = \int_a^b \left(g(y) \int_c^d f(x)\,dx\right) dy$ (since $g(y)$ is treated as a constant in the inner integral) $=$ $\left(\int_c^d f(x)\,dx\right) \int_a^b g(y)\,dy$ (since $\int_c^d f(x)\,dx$ is a constant and can therefore be taken outside the integral).

$\int_0^1 \int_1^2 ye^x\,dx\,dy = \frac{1}{2}(e^2 - e)$ no matter how we compute it.

Chapter 15 Review

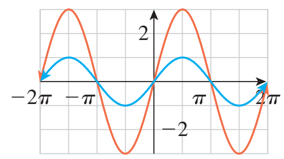
1. 0; 1; 0; $x^3 + x^2$; $x(y + k)(x + y + k - z) + x^2$
3. Reading left to right, starting at the top: 4, 0, 0, 3, 0, 1, 2, 0, 2
5. $f_x = 2x + y,\ f_y = x,\ f_{yy} = 0$ **7.** 0
9. $\frac{\partial f}{\partial x} = \frac{-x^2 + y^2 + z^2}{(x^2 + y^2 + z^2)^2},\ \frac{\partial f}{\partial y} = -\frac{2xy}{(x^2 + y^2 + z^2)^2},$ $\frac{\partial f}{\partial z} = -\frac{2xz}{(x^2 + y^2 + z^2)^2},\ \left.\frac{\partial f}{\partial x}\right|_{(0,1,0)} = 1$

11. Absolute minimum at (1, 3/2) **13.** Saddle point at (0, 0) **15.** Absolute maximum at each point on the circle $x^2 + y^2 = 1$ **17.** 1/27 at (1/3, 1/3, 1/3) **19.** $(0, 2, \sqrt{2})$ **21.** 4; $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$ **23.** Minimum value = 5 at (2, 1, 1/2) **25.** 2 **27.** ln 5 **29.** 4/5
31. a. $h(x, y) = 5000 - 0.8x - 0.6y$ hits per day (x = number of new customers at JungleBooks.com, y = number of new customers at FarmerBooks.com) **b.** 250 **33. a.** 2320 hits per day **b.** $0.08 + 0.00003x$ hits (daily) per dollar spent on television advertising per month; increases with increasing x **c.** \$4000 per month
35. About 15,800 orders per day **37.** \$23,050

Chapter 16

Section 16.1

1.



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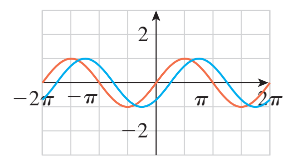

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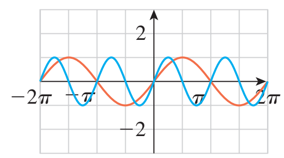

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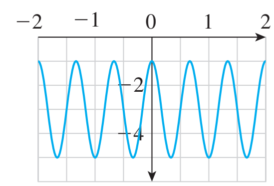

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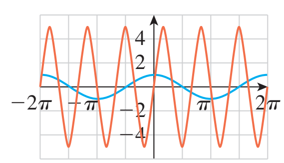

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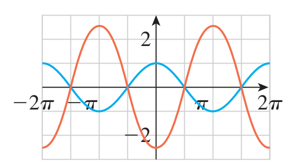
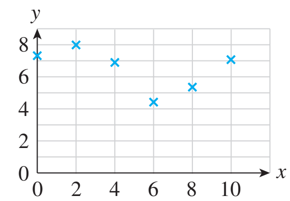
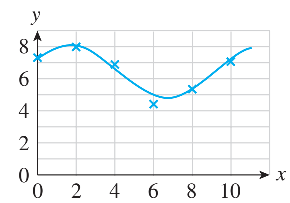


13. $f(x) = \sin(2\pi x) + 1$ **15.** $f(x) = 1.5\sin(4\pi(x - 0.25))$
17. $f(x) = 50\sin(\pi(x - 5)/10) - 50$ **19.** $f(x) = \cos(2\pi x)$
21. $f(x) = 1.5\cos(4\pi(x - 0.375))$
23. $f(x) = 40\cos(\pi(x - 10)/10) + 40$
25. $f(t) = 4.2\sin(\pi/2 - 2\pi t) + 3$
27. $g(x) = 4 - 1.3\sin[\pi/2 - 2.3(x - 4)]$ **31.** $\sqrt{3}/2$
37. $\tan(x + \pi) = \tan(x)$ **39. a.** $2\pi/0.602 \approx 10.4$ years.
b. Maximum: $58.8 + 57.7 = 116.5 \approx 117$; minimum: $58.8 - 57.7 = 1.1 \approx 1$
c. $1.43 + P/4 + P = 1.43 + 13.05 \approx 14.5$ years, or midway through 2011 **41. a.** Maximum sales occurred when $t \approx 4.5$ (during the first quarter of 1996). Minimum sales occurred when $t \approx 2.2$ (during the third quarter of 1995) and $t \approx 6.8$ (during the third quarter of 1996). **b.** Maximum quarterly revenues were \$0.561 billion; minimum quarterly revenues were \$0.349 billion. **c.** maximum: $0.455 + 0.106 = 0.561$; minimum: $0.455 - 0.106 = 0.349$ **43.** Amplitude = 0.106, vertical offset = 0.455, phase shift = $-1.61/1.39 \approx -1.16$ angular frequency = 1.39, period = 4.52. In 1995 and 1996, quarterly revenue from the sale of computers at Computer City fluctuated in cycles of 4.52 quarters about a baseline of \$0.455 billion. Every cycle, quarterly revenue peaked at \$0.561 billion (\$0.106 billion above the baseline) and dipped to a low of \$0.349 billion. Revenue peaked early in the middle of the first quarter of 1996 (at $t = -1.16 + (5/4) \times 4.52 = 4.49$).
45. $P(t) = 7.5\sin[\pi(t - 13)/26] + 12.5$
47. $s(t) = 7.5\sin(\pi(t - 9)/6) + 87.5$
49. $s(t) = 7.5\cos(\pi t/6) + 87.5$
51. $d(t) = 5\sin(2\pi(t - 1.625)/13.5) + 10$

53. **a.** $u(t) = 2.5 \sin(2\pi(t - 0.75)) + 7.5$
b. $c(t) = 1.04^t[2.5 \sin(2\pi(t - 0.75)) + 7.5]$
55. **a.** $P \approx 8,\ C \approx 6,\ A \approx 2,\ \alpha \approx 8$ (Answers may vary)

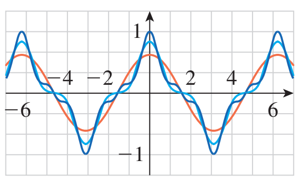


b. $C(t) = 1.755 \sin[0.636(t - 9.161)] + 6.437$

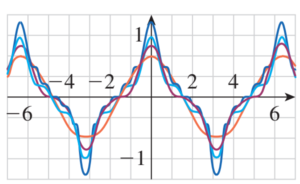


c. 9.9, 4.7%, 8.2%

57. **a.**

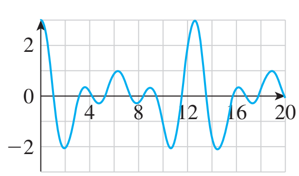


b. $y_{11} = \frac{2}{\pi}\cos x + \frac{2}{3\pi}\cos 3x + \frac{2}{5\pi}\cos 5x + \frac{2}{7\pi}\cos 7x + \frac{2}{9\pi}\cos 9x + \frac{2}{11\pi}\cos 11x$



c. $y_{11} = \frac{6}{\pi}\cos\frac{x}{2} + \frac{6}{3\pi}\cos\frac{3x}{2} + \frac{6}{5\pi}\cos\frac{5x}{2} + \frac{6}{7\pi}\cos\frac{7x}{2} + \frac{6}{9\pi}\cos\frac{9x}{2} + \frac{6}{11\pi}\cos\frac{11x}{2}$

59. The period is approximately 12.6 units



61. Lows: $B - A$; Highs: $B + A$ **63.** He is correct. The other trig functions can be obtained from the sine function by first using the formula $\cos x = \sin(x + \pi/2)$ to obtain cosine, and then using the formulas
$\tan x = \frac{\sin x}{\cos x},\ \cot x = \frac{\cos x}{\sin x},\ \sec x = \frac{1}{\cos x},\ \csc x = \frac{1}{\sin x}$
to obtain the rest. **65.** The largest B can be is A. Otherwise, if B is larger than A, the low figure for sales would have the negative value of $A - B$.

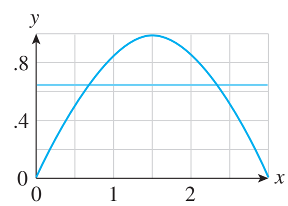
Section 16.2

1. $\cos x + \sin x$ **3.** $(\cos x)(\tan x) + (\sin x)(\sec^2 x)$
5. $-2 \csc x \cot x - \sec x \tan x + 3$ **7.** $\cos x - x \sin x + 2x$
9. $(2x - 1)\tan x + (x^2 - x + 1)\sec^2 x$
11. $-[\csc^2 x(1 + \sec x) + \cot x \sec x \tan x]/(1 + \sec x)^2$
13. $-2 \cos x \sin x$ **15.** $2 \sec^2 x \tan x$ **17.** $\pi \cos\left[\frac{\pi}{5}(x - 4)\right]$
19. $-(2x - 1)\sin(x^2 - x)$
21. $(2.2x^{1.2} + 1.2)\sec(x^{2.2} + 1.2x - 1)\tan(x^{2.2} + 1.2x - 1)$
23. $\sec x \tan x \tan(x^2 - 1) + 2x \sec x \sec^2(x^2 - 1)$
25. $e^x[-\sin(e^x) + \cos x - \sin x]$ **27.** $\sec x$
33. $e^{-2x}[-2 \sin(3\pi x) + 3\pi \cos(3\pi x)]$
35. $1.5[\sin(3x)]^{-0.5}\cos(3x)$
37. $\frac{x^4 - 3x^2}{(x^2 - 1)^2}\sec\left(\frac{x^3}{x^2 - 1}\right)\tan\left(\frac{x^3}{x^2 - 1}\right)$
39. $\frac{\cot(2x - 1)}{x} - 2 \ln|x| \csc^2(2x - 1)$
41. **a.** Not differentiable at 0 **b.** $f'(1) \approx 0.5403$
43. 0 **45.** 2 **47.** Does not exist **49.** $1/\sec^2 y$
51. $-[1 + y\cos(xy)]/[1 + x\cos(xy)]$
53. $c'(t) = 7\pi \cos[2\pi(t - 0.75)]$; $c'(0.75) \approx$ \$21.99 per *year* $\approx$ \$0.42 per week **55.** $N'(6) \approx -32.12$ On January 1, 2003, the number of sunspots was decreasing at a rate of 32.12 sunspots per year.
57. $c'(t) = 1.035^t[\ln(1.035)(0.8 \sin(2\pi t) + 10.2) + 1.6\pi \cos(2\pi t)]$; $c'(1) = 1.035[10.2 \ln|1.035| + 1.6\pi] \approx$ \$5.57 per year, or \$0.11 per week.
59. **a.** $d(t) = 5 \cos(2\pi t/13.5) + 10$
b. $d'(t) = -(10\pi/13.5)\sin(2\pi t/13.5)$; $d'(7) \approx 0.270$. At noon, the tide was rising at a rate of 0.270 feet per hour.
61. **a.** (III) **b.** Increasing at a rate of 0.157 degrees per thousand years **63.** -6; 6 **65.** Answers will vary. Examples: $f(x) = \sin x$; $f(x) = \cos x$ **67.** Answers will vary. Examples: $f(x) = e^{-x}$; $f(x) = -2e^{-x}$ **69.** The graph of $\cos x$ slopes down over the interval $(0, \pi)$, so that its derivative is negative over that interval. The function $-\sin x$, and not $\sin x$, has this property. **71.** The derivative of $\sin x$ is $\cos x$. When $x = 0$, this is $\cos(0) = 1$. Thus, the tangent to the graph of $\sin x$ at the point $(0, 0)$ has slope 1, which means it slopes upward at 45°.

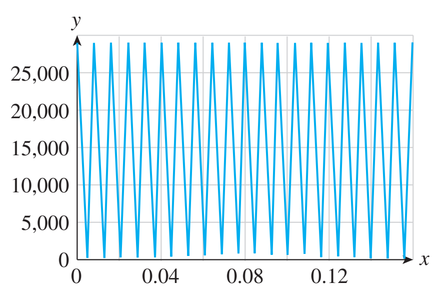
Section 16.3

1. $-\cos x - 2 \sin x + C$ **3.** $2 \sin x + 4.3 \cos x - 9.33x + C$
5. $3.4 \tan x + (\sin x)/1.3 - 3.2e^x + C$
7. $(7.6/3)\sin(3x - 4) + C$ **9.** $-(1/6)\cos(3x^2 - 4) + C$
11. $-2 \cos(x^2 + x) + C$ **13.** $(1/6)\tan(3x^2 + 2x^3) + C$
15. $-(1/6)\ln|\cos(2x^3)| + C$

17. $3 \ln|\sec(2x-4)+\tan(2x-4)|+C$
19. $(1/2)\sin(e^{2x}+1)+C$ **21.** -2 **23.** $\ln(2)$ **25.** 0
27. 1 **33.** $-\frac{1}{4}\cos(4x)+C$ **35.** $-\sin(-x+1)+C$
37. $[\cos(-1.1x-1)]/1.1+C$ **39.** $-\frac{1}{4}\ln|\sin(-4x)|+C$
41. 0 **43.** 2π **45.** $-x\cos x+\sin x+C$
47. $\left[\frac{x^2}{2}-\frac{1}{4}\right]\sin(2x)+\frac{x}{2}\cos(2x)+C$
49. $-\frac{1}{2}e^{-x}\cos x-\frac{1}{2}e^{-x}\sin x+C$ **51.** π^2-4
53. Average $=2/\pi$



55. Diverges **57.** Converges to $1/2$
59. $C(t)=0.04t+\frac{2.6}{\pi}\cos\left[\frac{\pi}{26}(t-25)\right]+1.02$
61. 12 feet **63.** 79 sunspots
65. $P(t)=7.5\sin[(\pi/26(t-13)]+12.5$; 7.7%
67. a. Average voltage over $[0, 1/6]$ is zero; 60 cycles per second.
b.



c. 116.673 volts. **69.** $50,000 **71.** It is always zero.
73. 1 **75.** $s=-\frac{K}{\omega^2}\sin(\omega t-\alpha)+Lt+M$ for constants L and M

Chapter 16 Review

1. $f(x)=1+2\sin x$
3. $f(x)=2+2\sin[\pi(x-1)]=2+2\sin[\pi(x+1)]$
5. $f(x)=1+2\cos(x-\pi/2)$
7. $f(x)=2+2\cos[\pi(x+1/2)]=2+2\cos[\pi(x-3/2)]$
9. $-2x\sin(x^2-1)$ **11.** $2e^x\sec^2(2e^x-1)$
13. $4x\sin(x^2)\cos(x^2)$ **15.** $2\sin(2x-1)+C$
17. $\tan(2x^2-1)+C$ **19.** $-\frac{1}{2}\ln|\cos(x^2+1)|+C$ **21.** 1
23. $-x^2\cos x+2x\sin x+2\cos x+C$
25. $s(t)=10{,}500+1500\sin[(2\pi/52)t-\pi]\approx 10{,}500+1500\sin(0.12083t-3.14159)$ **27.** $222,300

Appendix A

1. False statement **3.** Not a statement, because it is not a declarative sentence **5.** True statement **7.** True (we hope!) statement **9.** Not a statement, because it is self-referential. **11.** $(\sim p)\wedge q$ **13.** $(p\wedge r)\wedge q$ or just $p\wedge q\wedge r$ **15.** $p\vee(\sim p)$ **17.** Willis is a good teacher and his students do not hate math. **19.** Either Carla is a good teacher, or she is not. **21.** Willis' students both hate and do not hate math. **23.** It is not true that either Carla is a good teacher or her students hate math. **25.** F **27.** F **29.** T **31.** T **33.** T **35.** F **37.** T **39.** F **41.** T **43.** T **45.** T **47.** T

49.

p	q	$\sim q$	$p\wedge\sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

51.

p	$\sim p$	$\sim(\sim p)$	$\sim(\sim p)\vee p$
T	F	T	T
F	T	F	F

53.

p	q	$\sim p$	$\sim q$	$(\sim p)\wedge(\sim q)$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

55.

p	q	r	$p\wedge q$	$(p\wedge q)\wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

57.

p	q	r	$q\vee r$	$p\wedge(q\vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

59.

p	q	$q \vee p$	$p \rightarrow (q \vee p)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

61.

p	q	$p \vee q$	$p \leftrightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	T

63.

p	$p \wedge p$
T	T
F	F

same

65.

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

same

67.

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$(\sim p) \wedge (\sim q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

same

69.

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

same

71.

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$(\sim q) \rightarrow (\sim p)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

same

73. Contradiction **75.** Contradiction **77.** Tautology **79.** $(\sim p) \vee p$ **81.** $(\sim p) \vee \sim(\sim q)$ **83.** $(p \vee (\sim p)) \wedge (p \vee q)$ **85.** Either I am not Julius Caesar or you are no fool. **87.** It is raining and I have forgotten either my umbrella or my hat. **89.** Contrapositive: "If I do not exist, then I do not think."Converse: "If I am, then I think." **91.** (A) It is the contrapositive of the given statement.

93. $h \rightarrow t$
$\underline{h}$
$\therefore t$
Valid; Modus Ponens

95. $r \rightarrow u$
$\underline{\sim r}$
$\therefore \sim u$
Invalid

97. $g \rightarrow m$
$\underline{\sim m}$
$\therefore \sim g$
Valid: Modus Tollens

99. $m \vee b$
$\underline{\sim b}$
$\therefore m$
Valid: Disjunctive Syllogism

101. $s \vee a$
$\underline{a}$
$\therefore \sim s$
Invalid

103. John is green. **105.** John is not a swan. **107.** He is a gentleman. **109.** Their truth tables have the same truth values for corresponding values of the variables. **111.** A and B are both contradictions. **113.** Answers may vary. Let p: "You have smoker's cough," and q: "You smoke." **115.** Let p: "It is summer in New York," and q: "It is summer in Seattle."