Groups of three (or two) students each will work on one of the final projects listed below. You must work with other students than the ones you worked with for the student presentation.

Each group will prepare a complete written solution (approximately five pages) and a 15-minute presentation. The paper does not need to be typeset if the handwriting is legible. The projects will be presented during the final exam period on **Thursday**, **December 8**, 1:00 - 3:45. The accompanying papers are due before the start of the presentations.

The student group will be graded as a group. All group members must contribute to both the written solution and the presentation in equal parts. If members of a group feel that one member is not contributing in a meaningful way, they can ask me to remove the particular student from their group.

The group will be graded foremost on the mathematical correctness and mathematical clarity of their solution. Other criteria include the quality and completeness of the written report, the quality of the group presentation, making effective use of the allotted time, and staying within the time frame of 15 minutes for the oral presentation.

**Projects** (The numbers refer to end-of-chapter projects.)

- 1. Quaternary system State and prove theorems for numbers in base 4 corresponding to the theorems in Section 2.1.3.
- 2. Countability of algebraic numbers (2.2).
- 3. Stereographic projection (2.8).
- 4. An introduction to quaternions (definition, interpretation as complex  $2 \times 2$  matrices, non-commutativity, conjugates, inverses).
- 5. Leibniz segments (3.1 and 3.2). [Requires knowledge of geometry software such as *Geometer's Sketchpad* or *GeoGebra*.]
- 6. nth differences and polynomial functions (3.5; see also Section 3.3.2).
- 7. Limit definitions for the number e (3.6).
- 8. How likely are real and rational solutions for quadratic equations with integer coefficients? (4.2).
- 9. Cardano-Tartaglia method for solving cubic equations (2.5).
- 10. Ferrari's method to solve quartic equations (4.5).
- 11. Newton's method (4.6). Include a discussion of the application of Newton's method to the polynomial  $f(x) = x^2 + 1$ . [Requires use of appropriate software, *e.g.*, *Excel* or *Mathematica*.]