# Using Prediction and Clickers to Address Students' Mathematical Misconceptions

**Kien Lim**

**Dept. of Math Sciences**

**March 10, 2011**

# Outline of Presentation

1. Experience for Yourself
2. Benefits of Using Prediction & Clickers
3. Classroom Research
4. Results and Discussion

# 1. Experience for Yourself

Fill in the blank with either >, <, or =

$$\frac{81405}{67092} \div \frac{2884}{3717} \square \frac{81405}{67092}$$

A. >

B. <

C. =

# 1. Experience for Yourself

If $N$ is a natural number,
then $N$ can take any of these values: 1, 2, 3, 4, 5, ….

Is the following inequality always true, sometimes true, or never true?

N is a natural number: $\frac{67}{89} \times N < N$

A. Always True (AT)

B. Sometimes True (ST)

C. Never True (NT)

Item 2

# 1. Experience for Yourself

## Two Misconceptions

- Multiplication Makes Bigger (MMB)
- Division Makes Smaller (DMS)

MMB-DMS can affect students' thinking

A piece of cheese weighs 0.923kg. 1 kg costs 27.50 kr. Find out the price of the cheese. Which operation would you have to perform?

$27.50 + 0.923$ $27.50 \div 0.923$ $0.923 \times 27.50$ $27.50 - 0.923$

29%

(Ekenstam and Greger, 1983)

# 1. Experience for Yourself

## Two Misconceptions

- Multiplication Makes Bigger (MMB)
- Division Makes Smaller (DMS)

MMB-DMS can affect students' thinking.

A piece of cheese weighs 0.923kg. 1 kg costs 27.50 kr. Find out the price of the cheese. Which operation would you have to perform?

$27.50 + 0.923$ $27.50 \div 0.923$ $0.923 \times 27.50$ $27.50 - 0.923$

A common explanation: "The piece of cheese must cost less than 27.50, so I must *divide to get a smaller number*" (p. 376)

(Ekenstam and Greger, 1983)

# Benefits of Using Prediction

- Allows students to rely on intuition or capitalize on certain structural properties, instead of performing procedures (Lim et al., 2010)
- Improves student understanding via cognitive conflict (Gunstone et al., 1992; Palmer, 1995; Lavoie, 1999)
- Helps students notice certain structures in math (Buendía & Cordero, 2005; Fischbein & Grossman, 1997; Lim, 2006)
- Increases students' level of engagement (White & Gunstone, 1992; Kim & Kasmer, 2007)

"The commitment involved in deciding on a prediction can have powerful motivation effects." (White & Gunstone, 1992, p. 63)

# Benefits of Using Clickers

- Requires students to participate actively
- Provides a safe environment
- Provides immediate feedback
- Facilitates class discussion/debate
- Creates a fun atmosphere

(Cline, Zullo, & Parker, 2006)

# 3. Classroom Research

## Participants

- Prospective teachers in 4-8 grade band
- Two sections of MATH 3308
  (Proportional & Algebraic Reasoning 1)

# 3. Classroom Research

## Research Objectives

- to investigate the viability of a lesson using five such items in addressing the MMB-DMS misconceptions
- to compare students' pre-post improvement between two groups

Prediction-clicker-discussion

1. Read the question
2. Write one's prediction
3. Vote via clickers
4. Discuss in small groups
5. Re-vote
6. See explanation

Discussion-only

1. Read the question
4. Discuss in small groups
6. See explanation

Item 2

Is the following inequality always true, sometimes true, or never true?

N is a natural number: $\frac{67}{89} \times N < N$

A. Always True (AT)

B. Sometimes True (ST)

C. Never True (NT)

Item 2

Many children will choose "NT". Why?

N is a natural number: $\frac{67}{89} \times N < N$

Because they think "multiplication makes bigger".

For example, $6 \times 10 < 10$ is false (In this case, $N = 10$).

$6 \times 10 = 10 + 10 + 10 + 10 + 10 + 10 = 60$.

So "6 times 10 makes 10 bigger".

Item 2

Note: $\frac{67}{89}$ is a proper fraction. i.e, $\frac{67}{89} < 1$.

The correct answer is "AT". Why?

N is a natural number: $\frac{67}{89} \times N < N$

Suppose N = 10. $\frac{67}{89} \times 10 \ ? \ 10$

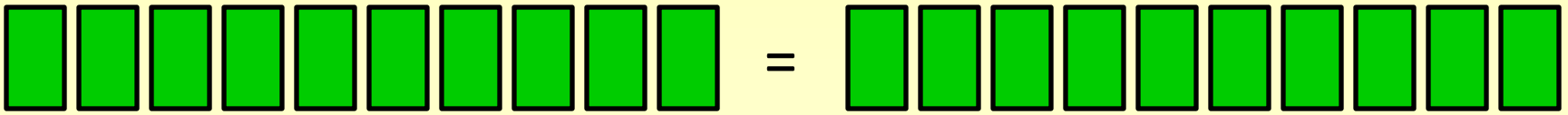
Item 2

Note: $\frac{67}{89}$ is a proper fraction. i.e, $\frac{67}{89} < 1$.

The correct answer is "AT". Why?

N is a natural number: $\frac{67}{89} \times N < N$

Suppose N = 10. $1 \times 10 = 10$



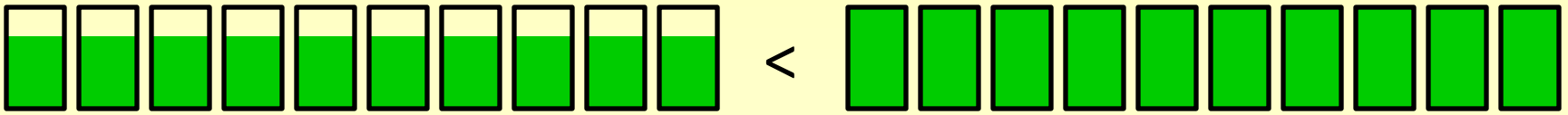
Item 2

Note: $\frac{67}{89}$ is a proper fraction. i.e, $\frac{67}{89} < 1$.

The correct answer is "AT". Why?

N is a natural number: $\frac{67}{89} \times N < N$

Suppose N = 10. $\frac{67}{89} \times 10 < 10$



Item 2

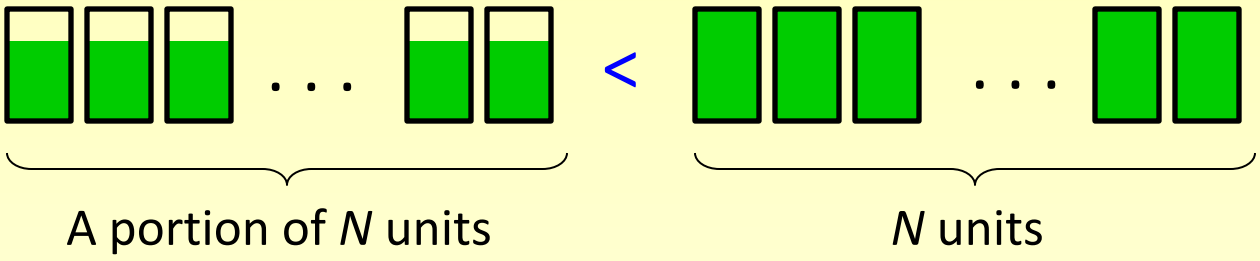
Note: $\frac{67}{89}$ is a proper fraction. i.e, $\frac{67}{89} < 1$.

$\frac{67}{89} \times N$ is actually a fractional part of the quantity $N$.

The correct answer is "AT". Why?

N is a natural number: $\frac{67}{89} \times N < N$

In general, $\frac{67}{89} \times N < N$



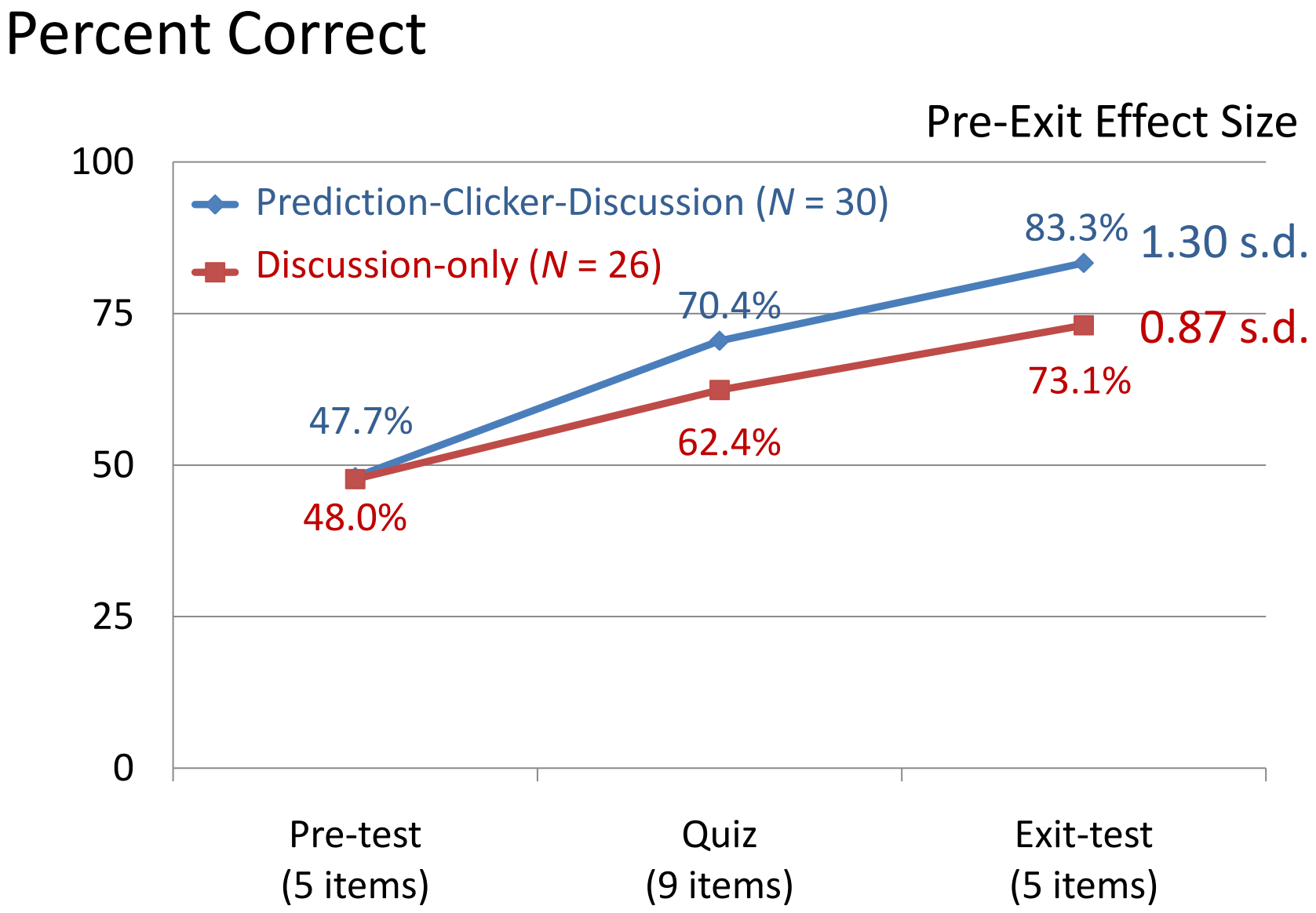

A proper fraction times a number makes the number smaller.

# 3. Classroom Research

## Data Sources

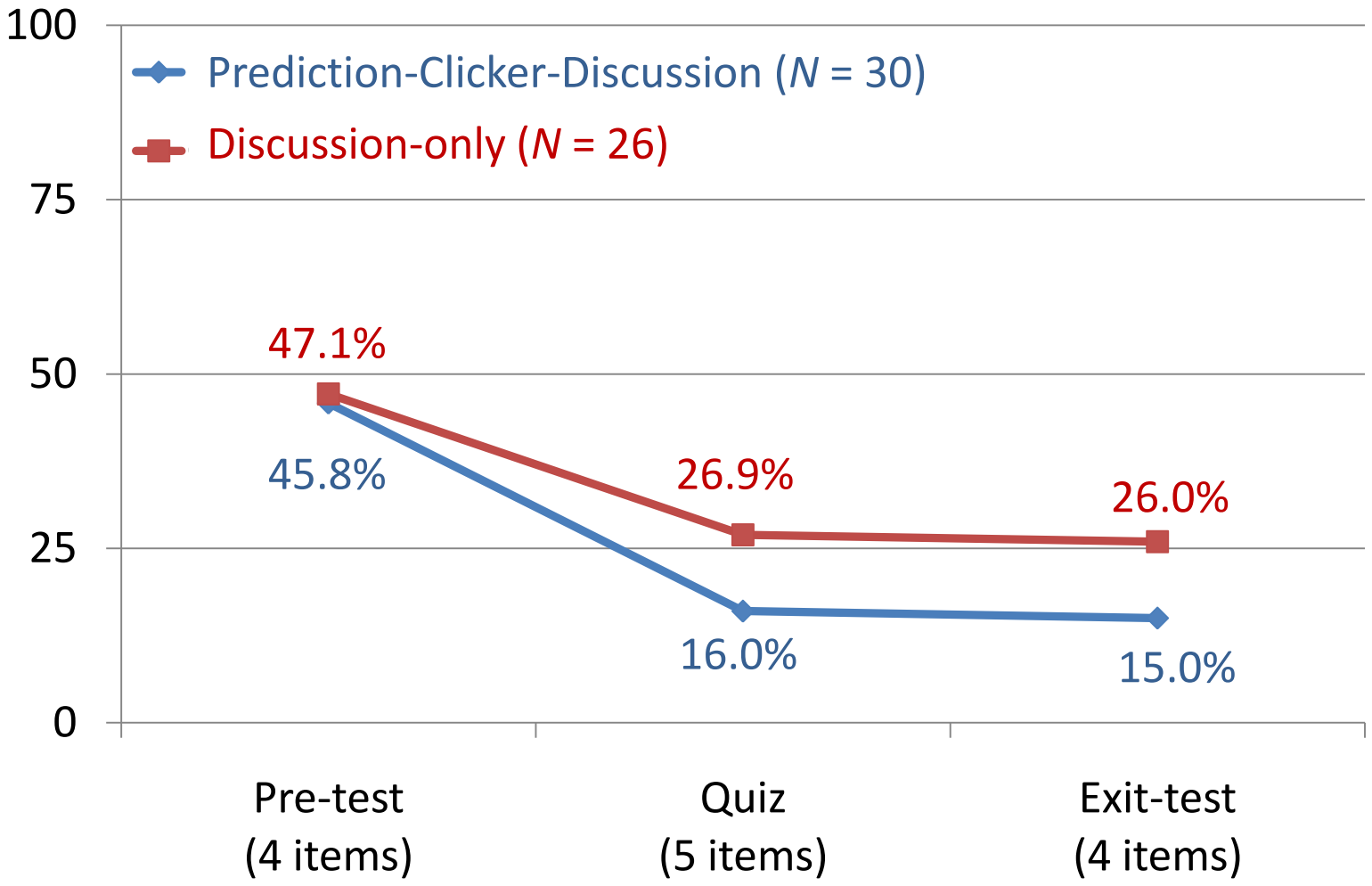
- Pre-test
  Post-lesson quiz
  Exit-test
- Students' written responses during the lesson
  Electronic data collected via clickers for one class
- Students' reflection logs
  1. Three important things that you have learned
  2. What do you like or dislike about the activity?
  3. What was the most effective part of the activity?
     What was the least effective part of the activity?
  4. Suggestions to make this activity more effective

# 4. Results and Discussion



# 4. Results and Discussion

## Percent of MMB-DMS-related Responses



# 4. Results and Discussion

## A particular student's response in the pre-test

3. Given that $p$ is a positive **proper fraction**, i.e. $0 < p < 1$.

$\frac{98765432}{12345678} \times p$ [ > ] $\frac{98765432}{12345678}$

Explain your reasoning.

Right side is multiplying itself by a number bigger than zero but less than 1, either way its bigger

## The same student's response in the exit-test

3. Given that $p$ is a positive **proper fraction**, i.e. $0 < p < 1$.

$\frac{98765432}{12345678} \times p$ [ < ] $\frac{98765432}{12345678}$

Explain your reasoning.

Multiplicating, especially w/ proper fractions doesnt always make answer bigger, but smaller

# 4. Results and Discussion

| **Things that Students Wrote about What They Had Learned** | P-C-D Class | D-only Class |
|---|---|---|
| *MMB-DMS misconceptions* | 21 | 16 |
| *Effect of proper/improper fractions on multiplication/division* | 8 | 13 |
| *Improper fractions being greater than 1* | 2 | 5 |
| *Plugging-in numbers for variables* | 5 | 2 |
| *Being cautious* | 3 | 3 |
| *Effect of multiplicative identity on multiplication* | 1 | 4 |
| *Dividing by a number as multiplying by its reciprocal* | 2 | 3 |

# 4. Results and Discussion

| **Things that Students Wrote about What They Liked** | P-C-D Class | D-only Class |
|---|---|---|
| *Learning certain mathematical ideas* | 11 | 8 |
| *Having Fun* | 6 | 2 |
| *Clickers* | 5 | 0 |
| *Thinking about the problems* | 4 | 5 |
| *Working in groups* | 4 | 4 |
| *Predicting* | 3 | 1 |
| *Learning from errors* | 3 | 0 |

# 4. Results and Discussion

## Responses about Most Effective Part of the Activity

- "It was the clickers. It will make everyone want to participate."
- "The part that give a prediction …"
- "Try to answer the problems by just looking at the problem without working it out. It really makes you think"
- "The PowerPoint Presentation"
- "The animation. This helps illustrate the solutions."

# An Article in Press

Lim, K. H. (in press). Addressing the multiplication makes bigger and division makes smaller misconceptions via prediction and clickers. *International Journal of Mathematical Education in Science and Technology*.

Questions?

# Thank You