How to Help Students Progress from Impulsive Disposition to Analytic Disposition?

5-min Group Discussion

5-min Write-up

5-min Gallery Walk

Pedagogical Suggestions

1. Use problem-based learning

Problem-based learning is a teaching method that "consists of carefully designed problems that challenge students to use problem solving techniques, self-directed learning strategies, team participation skills, and disciplinary knowledge"

(Center for Research in Teaching and Learning)

Pedagogical Suggestions

1. Use problem-based learning

How?

One possible approach

- Teacher poses a meaningful problem
- Students work individually
- Students discuss in small group
- Students present solutions
- Teacher orchestrates whole-class discussion, and highlights key concepts and useful habits of mind

Let's try problem-based learning now!

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
When candle A had burned 20 mm, candle B had burned 12 mm.
When candle B had burned 30 mm, how many mm would candle A have burned?

a. Solve this problem?

b. What key mathematical understandings do you want your students learn from working on this problem?

c. What habits of mind do you want your students to develop from working on this problem?

Think Pair Share

Let's try problem-based learning now!

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
When candle A had burned 20 mm, candle B had burned 12 mm.
When candle B had burned 30 mm, how many mm would candle A have burned?

2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
When candle P had burned 16 mm, candle Q had burned 10 mm.
When candle Q had burned 35 mm, how many mm would candle P have burned?

a. Solve this problem?

b. Structurally, how is this problem different from the Candle A-B problem?

Think Pair Share

Visualizing and Drawing Diagrams

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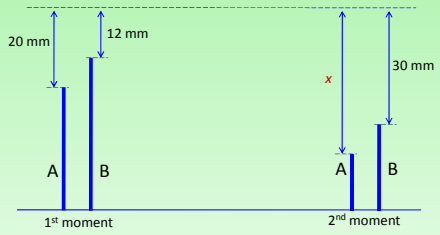
When candle A had burned 20 mm, candle B had burned 12 mm.
When candle B had burned 30 mm, how many mm would candle A have burned?



Pedagogical Suggestions

1. Use problem-based learning
2. Include superficially-similar-structurally-different problems
3. Encourage visualizing and drawing diagrams
4. Emphasize quantitative reasoning
   a. Focus on quantities
   b. Focus on relationships among quantities
   c. Focus on meanings of symbols and numbers

a. Focus on Quantities

1. Two identical candles, A and B, lighted at different times were burning at the same constant rate.
When candle A had burned 20 mm, candle B had burned 12 mm.
When candle B had burned 30 mm, how many mm would candle A have burned?

Length of candle A burned at the 1st moment

Length of candle B burned at the 1st moment

Length of candle A burned at the 2nd moment

Length of candle B burned at the2nd moment

List the quantities.

20 , 12 , and 30
20mm, 12mm, and 30mm

b. Focus on Relationships among Quantities

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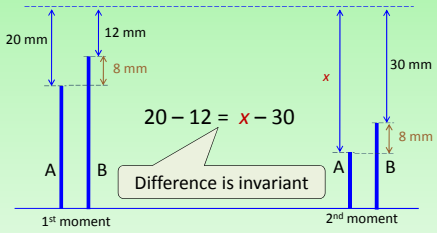

c. Focus on Meanings of Symbols and Numbers

2. Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates.
When candle P had burned 16 mm, candle Q had burned 10 mm.
When candle Q had burned 35 mm, how many mm would candle P have burned?

What does 1.6 represent?

$\frac{16}{10} = \frac{x}{35}$

Why is x/16 equal to 3.5?

For every 1mm candle Q burn, candle P burned 1.6 mm.

Candle P is burning 1.6 times as fast as candle Q.

c. Focus on Meanings of Symbols and Numbers

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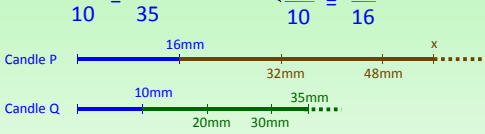
What does 3.5 represent?

Why is x/16 equal to 3.5?

$\frac{16}{10} = \frac{x}{35}$ $\quad \frac{35}{10} = \frac{x}{16}$



Comments from My Students (pre-service 4-8 teachers)

- "I learned to analyze the problem instead of rushing into a procedure, I used to do that."
- "This class helped me … by thinking deeper about that problem instead of just looking at the numbers and wanting to do something with them."
- "In this class, the concepts remain the same, yet the problems themselves are always quite different. I can no longer rely on 'similar problems' in order to figure out my homework or pass [the] exams."
- "This class is very demanding because I have to dedicate more time to learn how to get rid of those 'bad habits' that I have learned in previous classes."

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5. Avoid teaching algorithms prematurely

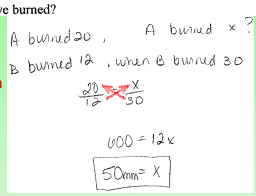
Avoid Teaching Algorithms Prematurely

Two identical candles, A and B, lighted at different times were burning at the same constant rate. When candle A had burned 20 mm, candle B had burned 12 mm. When candle B had burned 30 mm, how many millimeters would candle A have burned?

1° Align info

2° Set up a proportion

3° Cross-multiply



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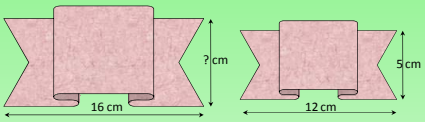
6. Assess conceptual understanding

Assess Conceptual Understanding



The original picture of a ribbon is shrunk proportionally …
What is the ratio of the breadth of the ribbon in the original picture (left) to the width of the ribbon in the new picture (right)?

19% 38% 0% 16% 28%

(a) 4 : 3 (b) 5 : $6\frac{2}{3}$ (c) 5 : 9 (d) 9 : 5 (e) None of the above

$\frac{16\text{ cm}}{12\text{ cm}} = \frac{x\text{ cm}}{5\text{ cm}} \Rightarrow x = 6\frac{2}{3}$ But $6\frac{2}{3} : 5$ is not among the choices.

6 out of 32 students chose (a).

Only 2 students chose (a) without any computation.

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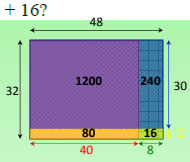
6. Assess conceptual understanding
7. Use contra problems in assessments

Use Contra Problems in Assessments (Teach A but Assess A')

An In-class Item

48 × 32 can be solved by finding the value of (40 + 8) × (30 + 2).

a. Use the area of a rectangle to show why (40 + 8) × (30 + 2) is equal to 1200 + 80 + 240 + 16?



An Mid-Term Exam

Use the area of a rectangle to show that 407 × 20 is the same as 8000 + 140.