Section 2.4

Complex numbers

If a and b are real numbers, the number $a + bi$ is a complex number, and it is said to be written in standard form. If $b = 0$, the number $a + bi = a$ is a real number. If $b \neq 0$, the number $a + bi$ is called an imaginary number. A number of the form bi, where $b \neq 0$, is called a pure imaginary number.

Equality of Complex Numbers

Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other $a + bi = c + di$ if and only if $a = c$ and $b = d$.

Principal Square Root of a Negative Number

If a is a positive number, the principal square root of the negative number $-a$ is defined as $\sqrt{-a} = \sqrt{a}i$.

Complex Conjugates

The numbers of the form $a + bi$ and $a - bi$ are called complex conjugates.

Problem 1. Write the complex number in the standard form $a + bi$.

a) $\sqrt{-9}$

b) $2 + \sqrt{-12}$

c) $5 + \sqrt{-4}$

d) i, i^2, i^3, i^4, i^5

e) $-6i^2 + 3i$

Problem 2. Perform the operation and write the result in the standard form.

a) $(-2 + 6i) + (13 - 7i)$

b) $(4 - 8i) - (6 + 9i)$

c) $\left(-3 + \sqrt{-24}\right) - \left(4 + \sqrt{2}i\right)$

d) $(3 - 4i)(2 + 5i)$

e) $(1 - 3i)^2 - (1 + 3i)^2$

Problem 3. Write the quotient in standard form.

a) $\frac{-22}{2i}$

b) $\frac{-3+2i}{4-i}$

c) $\frac{3i}{(2-3i)^2}$

Problem 4. Perform the operation and write the result in standard form.

a) $\frac{2i}{3+i}+\frac{4}{3-i}$

b) $\sqrt{-6}\cdot\sqrt{-8}$

c) $\left(\sqrt{-2}\right)^7$

Problem 5. Solve the quadratic equation.

a) $x^2+4x+8=0$

b) $4x^2-4x+37=0$

c) $x^2+x+1=0$