M-1312

Thursday, December 5, 2019 7:24 AM

#5 FIND THE INDEFINITE INTEGRAL OF

$$\int (2x+1)e^{3x}\,dx.$$

Integration by Parts!

Let u be chosen according to

I
L
A → Algebraic $(2x+1)$.
T
E → exponential (last priority for u)

and dv whatever remains.

$u = 2x+1 \Rightarrow du = 2\,dx$

$dv = e^{3x} \Rightarrow v = \int e^{3x}\,dx = \frac{1}{3}e^{3x}$ By u-substitution $u=3x$

Int. by Parts formula: $\int u\,dv = uv - \int v\,du$

Putting it all together:

$$\int (2x+1)e^{3x}\,dx = (2x+1)\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\,2\,dx$$

$$= 2x\frac{e^{3x}}{3} + \frac{e^{3x}}{3} - \frac{2}{3}\int e^{3x}\,dx$$

$$= \frac{2xe^{3x}}{3} + \frac{e^{3x}}{3} - \frac{2}{3}\left(\frac{e^{3x}}{3}\right)$$

$$\int (2x+1)e^{3x}\,dx = \frac{2}{3}xe^{3x} + \frac{e^{3x}}{9} + C \quad \boxed{\text{Answer}}$$

#7 CONSIDER THE REGION BOUNDED BY:

$$y = 1 - \sqrt{x^3}, \quad \text{X-AXIS}$$

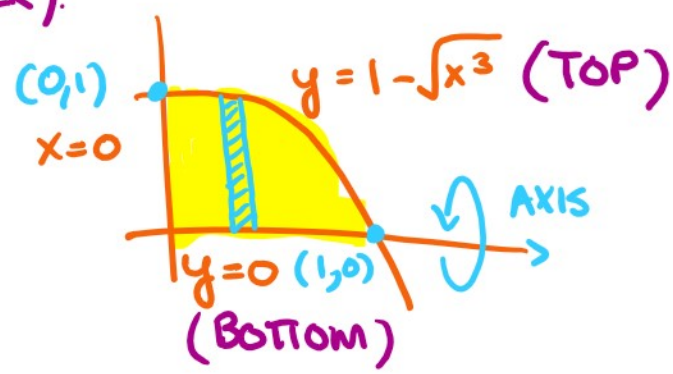
AND Y-AXIS.

FIND THE VOLUME OF SOLID OBTAINED BY ROTATING THIS REGION ABOUT X-AXIS.

WASHER OR SHELL? SEE THE AXIS OF ROTATION ! → X-AXIS

CURVES GIVEN INTERMS OF "x".

SO APPLY WASHER (EASIER).



Formula:

$$V = \pi \int_{x=a}^{b} (R^2 - r^2)\, dx$$

curves: $R(x) = 1 - \sqrt{x^3}$

$r(x) = 0$

Points of Intersection: $1 - \sqrt{x^3} = 0 \implies x = 0,\ x = 1.$

$$V = \pi \int_0^1 (1 - \sqrt{x^3})^2 - 0^2 \, dx$$

$$= \pi \int_0^1 (1 - 2\sqrt{x^3} + x^3)\, dx$$

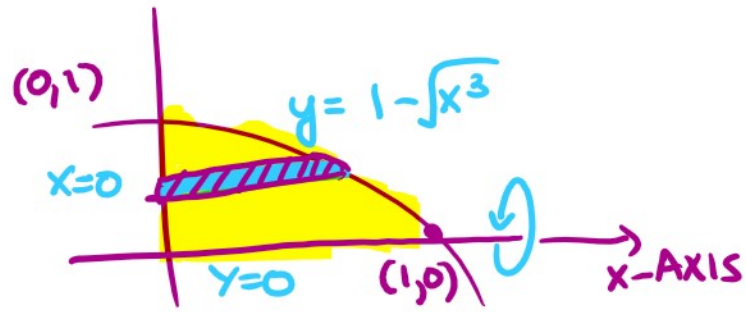
$$= \pi \left(x - 2\frac{x^{5/2}}{5/2} + \frac{x^4}{4} \right)\Bigg|_{x=0}^{1}$$

$$= \pi \left(1 - \frac{4}{\ } + \frac{1}{\ } \right) = \pi\left(1/5 + 1/4 \right) = 9\pi$$

$$= \pi\left(1 - \frac{4}{5} + \frac{1}{4}\right) = \pi\left(1/5 + 1/4\right) = \frac{9\pi}{20}$$

WHAT IF WE WANT TO USE SHELL METHOD?

Back to the Picture



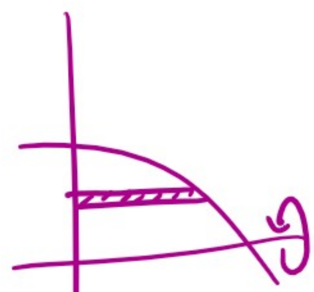
Convert all curves in terms of x = function of y.

$$y = 1 - \sqrt{x^3} \Rightarrow \sqrt{x^3} = 1 - y$$
$$\Rightarrow \left(\sqrt{x^3}\right)^2 = (1-y)^2$$
$$\Rightarrow x^3 = (1-y)^2$$
$$\Rightarrow x = (1-y)^{2/3}$$

and $x = 0$ is the other (left curve / Bottom)

POINTS OF INTERSECTION: $y = 0$, $y = 1$.

FORMULA: $V = 2\pi \int_{y=c}^{d} R(y)\, H(y)\, dy$



$R(y)$: Distance from axis of Rev. $= y$

$H(y)$: Height $(1-y)^{2/3}$

$$V = 2\pi \int_0^1 y\,(1-y)^{2/3}\,dy$$

$$= 2\pi \int^1 y\,(1-y)^{2/3}\,dy \quad \text{Hard to integrate!}$$

$$= 2\pi \int_{y=0} y\,(1-y)^{2/3}\,dy$$ Hard to integrate!

u-substitution! let $u = 1-y \Rightarrow du = -dy$

$y=0 \Rightarrow u=1$, $y=1 \Rightarrow u=0$.

$$V = 2\pi \int_{1}^{0} (1-u)\,u^{2/3}\,(-du)$$

$$= 2\pi \int_{1}^{0} \Big(-u^{2/3} + \underbrace{u * u^{2/3}}_{u^{1+2/3} = u^{5/3}}\Big)\,du$$

$$= 2\pi \left((-1)\frac{u^{5/3}}{5/3} + \frac{u^{8/3}}{8/3} \right)\Bigg|_{u=1}^{0}$$

$$= 2\pi * 0 - 2\pi * \left(-3/5 + 3/8\right)$$

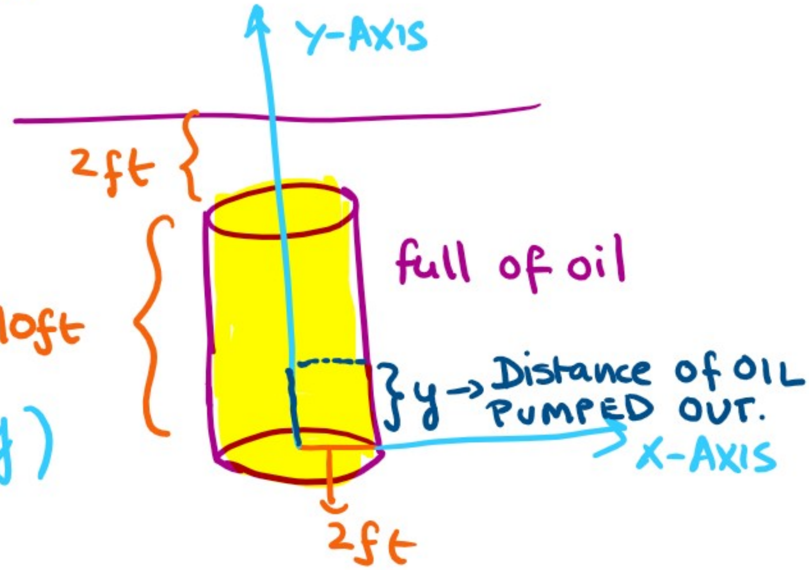
$$= 2\pi\left(\frac{3}{5}\right) - 2\pi\left(\frac{3}{8}\right)$$

$$= 6\pi * \frac{3}{40} = \frac{9\pi}{20}$$ same answer harder technique!

#3 A CYLINDRICAL TANK IS 10 feet HIGH WITH A DIAMETER OF 4 FEET. TANK IS BURIED UPRIGHT SO THAT THE TOP IS 2 FEET UNDERGROUND. THE TANK IS FULL OF OIL WITH A WEIGHT DENSITY OF 100 POUNDS PER CUBIC FOOT. HOW MUCH WORK IS

IS FULL OF OIL

POUNDS PER CUBIC FOOT. HOW MUCH WORK IS NEEDED TO PUMP ALL OF THE OIL FROM TANK TO GROUND LEVEL?



WORK DONE

$$= \int_{y=0}^{10} \underbrace{\Delta F}_{\text{Density} * \text{Volume}} * \underbrace{D(y)}_{(\text{TOTAL DISTANCE} - y)} \, dy$$

y ranges from 0 to 10 BECAUSE THAT IS THE JURISDICTION OF FLUID ACTING AS FORCE.

IF THE TANK WAS FILLED HALF WAY, THEN

$Y=0$ TO $Y = \frac{1}{2}(10) = 5$

IF THE TANK WAS FILLED 3/4 th THEN

$Y=0$ TO $Y = \frac{3}{4}(10) = 7.5.$

PUTTING IT ALL TOGETHER:

$$\text{WORK DONE} = \int_{y=0}^{10} \pi \underbrace{(100 * 2^2)}_{\Delta F} * \underbrace{(12-y)}_{D(y)} \, dy$$

$$= 400\pi \int_0^{10} (12-y)\,dy$$

$$= 4800\pi * 10 - 400\pi * \frac{10^2}{2}$$

$$= 4800\pi * 10 - 400\pi * \frac{10^2}{2}$$

$$= 48000\pi - 20{,}000\pi = \underbrace{28{,}000\pi}_{\text{ANSWER!}}$$

#8 WRITE THE SERIES IN SIGMA NOTATION. FIND THE SUM OF THE SERIES, IF POSSIBLE.

$$\underset{n=0}{-\frac{2}{3}} + \underset{n=1}{\frac{4}{9}} - \underset{n=2}{\frac{8}{27}} + \underset{n=3}{\frac{16}{81}} \ldots$$

$n=0 \Rightarrow a_0 = -2/3 \longrightarrow (-1)^{0+1}(2/3)^{0+1}$

$n=1 \Rightarrow a_1 = 4/9 \longrightarrow (-1)^{1+1}(2/3)^{1+1}$

$n=2 \Rightarrow a_2 = -8/27 \longrightarrow (-1)^{2+1}(2/3)^{2+1}$

$n=3 \Rightarrow a_3 = 16/81 \longrightarrow (-1)^{3+1}(2/3)^{3+1}$

$$\sum_{n=0}^{\infty} (-1)^{n+1} (2/3)^{n+1}$$

SUM = ?

$a + ar + ar^2 + ar^3 + \ldots$

IF $|r| < 1$ then SUM $= a/1-r$.

$a = -2/3$ (FIRST TERM) $\quad r = -2/3$

$$\text{SUM} = \frac{-2/3}{1-(-2/3)} = \frac{-2/3}{5/3} = -\frac{2}{5}.$$