Math 4329: Numerical Analysis Lecture 01

Natasha S. Sharma, PhD

What is Numerical Analysis?

- In the simplest sense, a computational extension of Calculus.
- Most of the Calculus problems will be analyzed from a computational point of view. This means we will study methods and algorithms to approximate the solution to these problems.
- To name a few problems...
  1. Evaluation of "complicated" functions at a point using "simpler" functions;
  2. Numerical Differentiation and Integration;
  3. Finding the x-intercepts of a function.
- Practical Implications:
  1. Evaulate the quality of the algorithm in terms of efficiency and accuracy.
  2. Use MATLAB software to solve these problems.

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Evaluation of "complicated" functions at a point using "simpler" functions

Problem: What is the value of e^0, e^{-1}, $e^{-0.5}$?

Solution:

- Use the simpler function Taylor Polynomial to find these values. This has to be based on a evaluation at a known point for example at 0 since we know $e^0 = 1$.
- Error in Taylor Polynomial
- Practice Problems for you.

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Taylor Polynomial

Taylor series is a representation of a function $f(x)$ as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point a.

- $f(x) = f(a)/0! + f'(a)(x-a)/1! + f''(a)\frac{(x-a)^2}{2!} + \cdots$
- Short Hand Infinite Series Form: $\sum_{n=0}^{\infty} f^n(a)\frac{(x-a)^n}{n!}$
- Example: Taylor Series for $f(x) = e^x$ at $a = 0$ is

$$f(x) = \frac{x^0}{0!} + \frac{x^1}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Idea: Take $x = -1, -0.5$ and obtain approximations to e^x using the "finite" Taylor series at $a = 0$.

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Finite Taylor Series: Taylor Polynomial

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- Example Taylor Polynomial of Degree 1 at $a = 0$ is

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Evaluate e^{-1} and $e^{-0.5}$

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Error in Taylor Polynomial

- Error = True Value - Approximated Value

- $e(x) = f(x) - p_n(x)$
  where,
  $e(x)$ denotes the error at x
  $f(x)$ is the function at x,
  $p_n(x)$ denotes the degree n polynomial
- Problem is we do not know $f(x)$?
- Error Representation Formula needed!

$$f(x) - p_n(x) = \frac{(x-a)^{n+1}}{(n+1)!} f^{(n+1)}(c_x),$$

c_x an unknown point between a and x.

Error in Taylor Polynomial

- Error = True Value - Approximated Value

- $e(x) = f(x) - p_n(x)$
  where,
  $e(x)$ denotes the error at x
  $f(x)$ is the function at x,
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