Math 2313, Test II

Name _____

1. If $f(x,y,z) = ln(x^2y^3z^4)$,

   a. Find the gradient of f at $(2,3,4)$.
   answer: $(1,1,1)$

   b. Find the derivative of f at $(2,3,4)$ in the direction of the vector $< 0,1,1 >$.
   answer: $\frac{2}{\sqrt{2}}$

   c. In what direction is the directional derivative smallest (most negative), at the point $(2,3,4)$?
   answer: $(-1,-1,-1)$

   d. Find the equation of the tangent plane to the surface $f(x,y,z) = ln(27648)$ at $(2,3,4)$.
   answer: $x + y + z = 9$

   e. If $x(p,q) = 2p^2q, y(p,q) = 3pq^3, z(p,q) = 4pq$, find $\frac{\partial f}{\partial q}$ when $p = 1, q = 1$.
   answer: 15

2. A cylinder initially has radius $r = 5$ and height $h = 10$, then the radius is decreased to 4.8 and the height is increased to 10.1. Given that the volume is $V = \pi r^2 h$, calculate both

   a. The exact change in volume, ΔV, and

   answer: -54.34

b. The approximate change in volume, dV.

answer: -54.98

3. If $f(x, y) = sin(x)cos(y)$, check each of the following points to see if it is a critical point. If so, classify as a local minimum, local maximum, or saddle point. (Hint: first calculate $f_x, ..., D$ below)

$f_x = cos(x)cos(y)$

$f_y = -sin(x)sin(y)$

$f_{xx} = -sin(x)cos(y)$

$f_{yy} = -sin(x)cos(y)$

$f_{xy} = -cos(x)sin(y)$

$D = f_{xx} * f_{yy} - (f_{xy})^2 = sin^2(x)cos^2(y) - cos^2(x)sin^2(y)$

a. $x = 0, y = \pi/2$
answer: critical, $D = -1$, saddle point

b. $x = 0, y = 0$
answer: not critical

c. $x = -\pi/2, y = 0$
answer: critical, $D = 1, f_{xx} = 1$, minimum