Math 2313, Test II

Name _____

1. If $f(x, y, z) = e^{2x+2y-z^2}$,

    a. Find the gradient of f at $(1, 1, 2)$.
       answer: $(2, 2, -4)$

    b. Find the derivative of f at $(1, 1, 2)$ in the direction of the vector $< 0, 3, 4 >$.
       answer: -2

    c. In what direction is the directional derivative largest, at the point $(1, 1, 2)$?
       answer: $(2, 2, -4)$

    d. Find the equation of the tangent plane to the surface $f(x, y, z) = 1$ at $(1, 1, 2)$.
       answer: $2x + 2y - 4z + 4 = 0$

2. If $f(x, y) = e^{xy}$ find $f_{xx} + f_{yy}$.

    answer: $(x^2 + y^2)e^{xy}$

3. A cylinder initially has radius $r = 5$ and height $h = 8$, then the radius is increased to 5.1 and the height is decreased to 7.8. Given that the surface area is $A = 2\pi(r^2 + rh)$, calculate both

   a. The exact change in surface area, ΔA, and

   answer: 4.9637

   b. The approximate change in surface area, dA.

   answer: 5.0265

4. If $f(x, y) = x^3 - 6xy + y^3$, find all critical points and classify each as a local minimum, local maximum, or saddle point.

   answer: $(0, 0)$ is saddle point, $(2, 2)$ is local minimum.

5. If $(U_x, U_y, U_z) = (3, 11, -1)$ at the point $(-1, 0, 0)$, which has spherical coordinates $\rho = 1, \phi = \frac{\pi}{2}, \theta = \pi$, find U_θ at this point. For spherical coordinates,
$$
\begin{aligned}
x &= \rho \, sin(\phi)cos(\theta) \\
y &= \rho \, sin(\phi)sin(\theta) \\
z &= \rho \, cos(\phi)
\end{aligned}
$$
   answer: -11