

Math 2313, Test II

Name _____

1. If $u(x, y) = x + e^{x^2y}$, find:

a. $\frac{\partial u}{\partial y} =$

answer: $x^2 e^{x^2y}$

b. $u_{xy} =$

answer: $(2x + 2x^3y)e^{x^2y}$

2. If $f(x, y, z) = \sqrt{x^2 + y^2 + z}$,

a. Find the gradient of f at $(12, 4, 9)$.

answer: $(\frac{24}{26}, \frac{8}{26}, \frac{1}{26})$

b. Find the derivative of f at $(12, 4, 9)$ in the direction of the vector $\langle 1, 1, 2 \rangle$.

answer: 0.5338

c. In what direction is the directional derivative of f smallest (most negative), at $(12, 4, 9)$?

answer: $-\nabla f = (-\frac{24}{26}, -\frac{8}{26}, -\frac{1}{26})$

d. Find the equation of the tangent plane to the surface $f(x, y, z) = 13$ at $(12, 4, 9)$.

answer: $24x + 8y + z = 329$

3. If the temperature in space is given by $T = T(x, y, z)$ and the coordinates of your rocket are $x = x(t), y = y(t), z = z(t)$, where t is time, write a formula which gives the rate of change of temperature with respect to time, inside your rocket.

answer: $\frac{dT}{dt} = \frac{\partial T}{\partial x} \frac{dx}{dt} + \frac{\partial T}{\partial y} \frac{dy}{dt} + \frac{\partial T}{\partial z} \frac{dz}{dt}$

4. A cylinder initially has radius $r = 1$ and height $h = 1$, then the radius is increased to 1.01 and the height is decreased to 0.96. Given that the surface area is $A = 2\pi(r^2 + rh)$, use differentials to find the approximate change dA in area.

answer: -0.02π

5. Write U_x in terms of U_r and U_θ , given that $r = \sqrt{x^2 + y^2}, \theta = \arctan(y/x)$. (Hint: the derivative of $\arctan(t)$ is $\frac{1}{1+t^2}$)

answer: $U_x = \cos(\theta)U_r - \sin(\theta)U_\theta/r$