

Math 2313, Test II

Name _____

1. If $w(x, y) = x^3 + xy^2 + e^{xy}$, find:
 - a. $\frac{\partial w}{\partial y} =$
answer: $2xy + xe^{xy}$
 - b. $w_{xx} + w_{yy} =$
answer: $8x + (x^2 + y^2)e^{xy}$
2. If $f(x, y, z) = \ln(\sqrt{x^3 + y + z^2})$,
 - a. Find the gradient of f at $(1, 1, 1)$.
answer: $\langle \frac{3}{6}, \frac{1}{6}, \frac{2}{6} \rangle$
 - b. Find the derivative of f at $(1, 1, 1)$ in the direction of the vector $\langle 2, 2, -1 \rangle$.
answer: $\frac{1}{3}$
 - c. In what direction is the directional derivative largest, at the point $(1, 1, 1)$?
answer: $\langle \frac{3}{6}, \frac{1}{6}, \frac{2}{6} \rangle$
 - d. Find the equation of the tangent plane to the surface $f(x, y, z) = \ln(\sqrt{3})$ at $(1, 1, 1)$.
answer: $3x + y + 2z = 6$

3. If the temperature is $T(x, y) = x^2y^3 + \ln(xy)$, what is the rate of change of temperature in a car at $(1, 1)$, if the velocity of the car is $(\frac{dx}{dt}, \frac{dy}{dt}) = (2, 5)$?
answer: 26

4. A cylinder initially has radius $r = 1$ and height $h = 1$, then the radius is increased to 1.01 and the height is decreased to 0.96. Given that the volume is $V = \pi r^2 h$, use differentials to find the approximate change dV in volume.

answer: -0.02π

5. If $(U_x, U_y, U_z) = (3, 4, -1)$ at the point $(-1, 0, 0)$, which has spherical coordinates $\rho = 1, \phi = \frac{\pi}{2}, \theta = \pi$, find U_θ at this point. For spherical coordinates,

$$\begin{aligned}x &= \rho \sin(\phi) \cos(\theta) \\y &= \rho \sin(\phi) \sin(\theta) \\z &= \rho \cos(\phi)\end{aligned}$$

answer: -4