

## Math 2313, Test II

Name \_\_\_\_\_

1. a. If  $r = \sqrt{x^2 + y^2}$ ,  $\theta = \arctan(y/x)$ , find  $r_x$  and  $\theta_x$ . (Hint: the derivative of  $\arctan(t)$  is  $\frac{1}{1+t^2}$ )  
answer:  $r_x = x/\sqrt{x^2 + y^2}$ ,  $\theta_x = -y/(x^2 + y^2)$

- b. Find  $\theta_{xy}$ .  
answer:  $\theta_{xy} = (-x^2 + y^2)/(x^2 + y^2)^2$

- c. Use the chain rule to write  $U_x$  in terms of  $U_r, U_\theta$ .  
answer:  $U_x = U_r x/\sqrt{x^2 + y^2} + U_\theta(-y)/(x^2 + y^2)$

2. If  $f(x, y, z) = e^{3x+2y-z^2}$ ,

- a. Find the gradient of  $f$  at  $(2, 5, 4)$ .  
answer:  $\langle 3, 2, -8 \rangle$

- b. Find the derivative of  $f$  at  $(2, 5, 4)$  in the direction of the vector  $\langle 3, 12, 4 \rangle$ .  
answer:  $\frac{1}{13}$

- c. Find the equation of the tangent plane to the surface  $f(x, y, z) = 1$  at the point  $(2, 5, 4)$ .  
answer:  $3x + 2y - 8z + 16 = 0$

3. The height of a mountain is given by  $z = 5000 - 3x + 2(y - 20)^2$ .

- a. If you move from the point  $x = 10, y = 20$  to  $x = 9.8, y = 20.1$ , exactly how much does your altitude change?  
answer: 0.62

- b. Use the differential  $dz$  to approximate the change in altitude in part (a).  
answer: 0.6

- c. If you are at  $x = 10, y = 20$ , in what direction should you head if you want to go downhill as fast as possible?  
answer:  $-\nabla z = \langle 3, 0 \rangle$