Math 2313, Test II

Name _____

1. If $w(x,y) = x^3z + xy^2z + e^{xyz}$, find:

   a. $\frac{\partial w}{\partial x} =$
   answer: $3x^2z + y^2z + yze^{xyz}$

   b. $w_{xyz} =$
   answer: $2y + (x^2y^2z^2 + 3xyz + 1)e^{xyz}$

2. If $f(x,y,z) = ln(\sqrt{x+y+z^2})$,

   a. Find the gradient of f at $(1,1,1)$.
   answer: $< \frac{1}{6}, \frac{1}{6}, \frac{2}{6} >$

   b. Find the derivative of f at $(1,1,1)$ in the direction of the vector $< 2,2,-1 >$.
   answer: $\frac{1}{9}$

3. The height of a mountain is given by $z = 5000 - 2(x-5)^2 + 5(y-8)$.

   a. If you move from the point $x = 5, y = 8$ to $x = 5.2, y = 8.1$, exactly how much does your altitude change?
   answer: 0.42

   b. Use the differential dz to approximate the change in altitude in part (a).
   answer: 0.5

   c. If you are at $x = 5, y = 8$, in what direction should you head if you want to go downhill as fast as possible?
   answer: $-\nabla z =< 0, -5 >$

   d. Write the equation of the tangent plane to this surface, at $(5, 8)$.
   answer: $z = 5000 + 5(y-8)$

4. If $(U_x, U_y, U_z) = (2, -2, 1)$ at the point $(-3, 0, 0)$, which has spherical coordinates $\rho = 3, \phi = \frac{\pi}{2}, \theta = \pi$, find U_θ at this point. For spherical coordinates,

$$\begin{aligned} x &= \rho\, sin(\phi) cos(\theta) \\ y &= \rho\, sin(\phi) sin(\theta) \\ z &= \rho\, cos(\phi) \end{aligned}$$

   answer: 6