Math 2313, Test III

Name _____

1. Evaluate $\int_0^{\frac{1}{2}} \int_0^x \frac{1}{1-x^2} dy\ dx$

   answer: $ln(\frac{2}{\sqrt{3}})$

2. A solid cube, with edges of length 2, has a density that is equal to the square of the distance from one corner of the box, that is, $\rho(x, y, z) = x^2+y^2+z^2$. What is the mass of the cube? What is its average density?

   answer: $M = 32, \rho_{ave} = 4$

3. Reverse the order of integration: $\int_0^4 \int_{y^2/8}^{\sqrt{y}} f(x, y) dx dy$

   answer: $\int_0^2 \int_{x^2}^{\sqrt{8x}} f(x, y) dy dx$

4. a. Find the volume of the region below the surface $z = 18 - \frac{x^2}{2} - \frac{y^2}{2}$ and above the xy plane. (Hint: convert to polar coordinates)

   answer: 324π

   b. Find the surface area of this same surface, above the xy plane. (Hint: again, convert to polar coordinates)

   answer: $2\pi(37\sqrt{37} - 1)/3$

5. For the joint probability distribution function $p(x, y) = \frac{1}{\pi}e^{-x^2-y^2}$ write an integral (don't evaluate it) which expresses the probability that $y < 3$.

   answer: $\int_{-\infty}^{\infty}\int_{-\infty}^{3}\frac{1}{\pi}e^{-x^2-y^2}dydx$