Math 2313, Test III

Name _____

1. Find the point on the plane $z = 2 - 2x - y$ closest to $(1, 1, 2)$. Then prove that this point really minimizes the distance using the second derivative test.
   answer: $(0, \frac{1}{2}, \frac{3}{2})$
   $d_{xx}d_{yy} - d_{xy}^2 = 24 > 0$ and $d_{xx} > 0$ so it's a minimum
   (where $d = (x-1)^2 + (y-1)^2 + (-2x-y)^2$ is the square of the distance)

2. Find the mass of the solid $0 \leq x \leq 2, 0 \leq y \leq x, 0 \leq z \leq y$, if the density is $\rho(x, y, z) = x^3yz^2$

   answer: 512/135

3. Find the volume of the cone $z = 2 - \sqrt{x^2 + y^2}$ above the xy plane, using a double integral. (Hint: much easier in polar coordinates)

answer: $8\pi/3$

4. Find the surface area of the cone of problem 3, above the xy plane (ie, don't include the base of the cone), using a double integral.

answer: $4\sqrt{2}\pi$

5. Reverse the order of integration: $\int_0^2 \int_{x^3}^{4x} f(x, y)\ dy\ dx$

answer: $\int_0^8 \int_{y/4}^{y^{\frac{1}{3}}} f(x, y)\ dx\ dy$