Math 2313, Test III

Name _____

1. If $f(x, y) = x^3 - 3xy + y^3$, find all critical points and classify each as a local minimum, local maximum, or saddle point.

answer: $(0, 0)$ is saddle point, $(1, 1)$ is local minimum.

2. Reverse the order of integration in $\int_0^1 \int_y^1 cos(x^2) dx dy$, and evaluate the new integral.

answer: $sin(1)/2$

3. a. The rectangular solid $0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 1$ has a density of $\rho(x, y, z) = 4x^3$. What is the mass of the cube?

answer: $M = 162$

b. Find the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of this solid. (Hint: the values of two of the three coordinates of the center are obvious and need not be calculated.)

answer: $(2.4, 1.0, 0.5)$

4. Find the volume above the $z = 0$ plane and below the surface $z = e^{x^2+y^2}$, over the region $1 \leq x^2 + y^2 \leq 9$. (Hint: convert integral to polar coordinates)

answer: $\pi(e^9 - e^1)$

5. Find the surface area of the surface $f(x, y) = xy$, above the disk $x^2 + y^2 \leq 16$. (Hint: convert integral to polar coordinates)

answer: $\frac{2\pi}{3}(17\sqrt{17} - 1)$