

### Math 2313, Test III

Name \_\_\_\_\_

1. What value should  $a$  have so that  $p(x, y) = ae^{-\frac{x^2+y^2}{2\sigma^2}}$  is a joint probability density?

answer:  $a = \frac{1}{2\pi\sigma^2}$

2. Find the centroid of the semicircle  $0 < y < \sqrt{a^2 - x^2}$ . You can use the fact that the mass (= area, since the density is equal to one) of the semicircle is  $\pi a^2/2$ . (Hint: evaluate the integral in polar coordinates.)

answer:  $(0, \frac{4}{3\pi}a)$

3. If  $f(x, y) = x^3 - xy + y^3$ , find all critical points and classify each as a local minimum, local maximum, or saddle point.

answer:  $(0, 0)$  is saddle point,  $(1/3, 1/3)$  is local minimum.

4. Consider the integral  $\int_0^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} f(x, y) dy dx$

a. Reverse the order of integration.

answer:  $\int_{-4}^4 \int_0^{\sqrt{16-y^2}} f(x, y) dx dy$

b. Write the integral in polar coordinates.

answer:  $\int_{-\pi/2}^{\pi/2} \int_0^4 f(r \cos(\theta), r \sin(\theta)) r dr d\theta$

5. a. Find the volume of the region below the surface  $z = x^{3/2}$  and above the box  $0 < x < 4, 0 < y < 1$ .

answer: 12.8

b. Find the surface area of this surface, above the same box.

answer:  $\frac{8}{27}(10^{3/2} - 1) = 9.0734$