

Math 2313, Final

Name _____

1. Find a vector parallel to both of the planes $x + y + z = 10$ and $y + z = 7$.

answer: $\langle 0, -1, 1 \rangle$

2. Find the equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 9$ at $(1, -2, 2)$.

answer: $2x - 4y + 4z = 18$

3. Find $\frac{dU}{dp}$ at $(x, y, z) = (1, 1, 0)$ if $U = x^3 + \ln(xy) + e^{3yz}$, and at this point, $\frac{dx}{dp} = 2$, $\frac{dy}{dp} = 1$, $\frac{dz}{dp} = 3$.

answer: 18

4. Evaluate $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} e^{x^2+y^2} dy dx$

answer: $\frac{\pi}{2}(e^9 - 1)$

5. Find the directional derivative of $f(x, y) = e^{y^2} + \sin(xy) + \ln(x)$ at the point $(1, 0)$ in the direction of the vector $\langle 3, -4 \rangle$.

answer: -1

6. If $f(x, y) = x^3 - 6xy + y^3$, find all critical points and classify each as a local minimum, local maximum, or saddle point. (Second derivative test shown on board.)

answer: $(0, 0)$ is saddle point, $(2, 2)$ is local minimum.

7. Write an integral which, if evaluated (but don't evaluate), would give the mass of the tetrahedron in the first octant under the plane $4x + 2y + z = 8$, if the density is given by $\rho(x, y, z)$.

answer: $\int_0^2 \int_0^{4-2x} \int_0^{8-4x-2y} \rho(x, y, z) dz dy dx$