Math 2313, Final

Name _____

1. Find a vector parallel to both planes $x + 3z = 3$ and $2x - 2y + 3z = 9$.

   answer: $< 6, 3, -2 >$

2. Find the length of the curve with parametric equations $x(t) = cos(t) + t * sin(t), y(t) = sin(t) - t * cos(t), z(t) = \frac{1}{2}t^2$, from $t = 0$ to $t = 3$. (Hint: the distance travelled by a rocket from $t = 0$ to $t = 3$ is the integral of its speed from $t = 0$ to $t = 3$.)

   answer: $\frac{9}{2}\sqrt{2}$

3. Find the directional derivative of $f(x, y) = sin(xy + x^2 + y^2)$, at $(0, \sqrt{\pi})$ in the direction of the vector $< 3, -4 >$.
   answer: $\sqrt{\pi}$

4. Find $\frac{\partial U}{\partial q}$ *using the chain rule*, if

   $U = yx^2 + ln(xy)$
   $x = pq$
   $y = q/p$

   answer: $(2xy + \frac{1}{x})p + (x^2 + \frac{1}{y})(\frac{1}{p}) = 3pq^2 + \frac{2}{q}$

5. If $f(x, y) = x^3 - 12xy + y^3$ find all critical points and classify each as a maximum, minimum or saddle point.

   answer: $(4, 4)$ is a minimum, $(0, 0)$ is a saddle point

6. Evaluate $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{x^2+y^2} dxdy$ (Hint: change to polar coordinates.)

answer: $\frac{\pi}{2}(e-1)$

7. Find the mass of the cylinder $x^2 + y^2 \leq 9, 0 \leq z \leq 4$ whose density is given by $\rho(x, y, z) = x^2 + y^2 + z^2$.

answer: 354π