Math 2313, Final

Name _____

1. Consider the surface $x^3 ln(y) + ze^{xz} = 0$:

   a. Write the equation for the tangent plane at the point $(1, 1, 0)$.

   answer: $y + z = 1$

   b. Write the parametric equations for the normal line at $(1, 1, 0)$.

   answer: $(x, y, z) = (1, 1 + t, t)$

2. Find $\frac{\partial U}{\partial p}$, *using the chain rule,* if
   $U = x^3 + ln(xy) + e^{3yz}$
   $x = pq$
   $y = q/p$
   $z = p^2 + q^2$

   answer: $(3x^2 + \frac{1}{x})q + (\frac{1}{y} + 3ze^{3yz})(-\frac{q}{p^2}) + (3ye^{3yz})(2p)$

3. Find the length of the curve with parametric equations $x(t) = cos(t) + t * sin(t), y(t) = sin(t) - t * cos(t), z(t) = t^2$, from $t = 1$ to $t = 3$.

answer: $4\sqrt{5}$

4. Find the directional derivative of $f(x, y, z) = x^3 ln(y) + ze^{xz}$ at the point $(1, 1, 0)$ in the direction of the vector $< -3, 4, -12 >$.

answer: $\frac{-8}{13}$

5. Find the mass of the cylinder $x^2 + y^2 \leq 9, 0 \leq z \leq 4$ whose density is given by $\rho(x, y, z) = x^2 + y^2 + z^2$.

answer: 354π

6. If $f(x, y) = x^3 - 12xy + y^3$ find all critical points and classify each as a maximum, minimum or saddle point.

answer: $(4, 4)$ is a minimum, $(0, 0)$ is a saddle point

7. Evaluate $\int_0^1 \int_y^1 \sqrt{1 - x^2} dx\ dy$. (Hint: you will need to reverse the order of integration.)

answer: $\frac{1}{3}$