Math 2313, Final

Name _____

1. Find a vector perpendicular to both of the curves $(sin(t), t, cos(t))$ and $(t^2, t, 1)$ at $(0, 0, 1)$. (Hint: first find a tangent vector for each curve at this point.)

   answer: $(0, 0, 1)$

2. Find the length of the curve with parametric equations $x(t) = cos(t) + t * sin(t), y(t) = sin(t) - t * cos(t), z(t) = t^2$, from $t = 1$ to $t = 3$.

   answer: $4\sqrt{5}$

3. Find a critical point of $f(x, y) = 4x^2 + 4xy - 2y^2 + 8x - 5y - 4$ and classify it as a local maximum, minimum or saddle point.

   answer: $(-0.25, -1.5)$ is a saddle point.

4. Evaluate $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{x^2+y^2} dydx$

   answer: $\frac{\pi}{2}(e^4 - 1)$

5. Find the directional derivative of $f(x, y, z) = x^4 + ln(xy^2) + e^{3yz}$ at the point $(1, 1, 0)$ in the direction of the vector $< 2, 1, -2 >$.

answer: 2

6. Write an integral which, if evaluated (but don't evaluate), would give the mass of the tetrahedron in the first octant under the plane $6x + 2y + z = 12$, if the density is given by $\rho(x, y, z) = x^3 + z^3$.

answer: $\int_0^2 \int_0^{6-3x} \int_0^{12-6x-2y} (x^3 + z^3) dz dy dx$

7. Given that $r = \sqrt{x^2 + y^2}, \theta = arctan(y/x)$, and that the derivative of $arctan(t)$ is $\frac{1}{1+t^2}$

   a. Write U_x in terms of U_r and U_θ.

   answer: $U_x = U_r(x/r) + U_\theta(-y/r^2)$

   b. Write U_y in terms of U_r and U_θ.

   answer: $U_y = U_r(y/r) + U_\theta(x/r^2)$

   c. (extra credit) Write $U_x^2 + U_y^2$ in terms of U_r and U_θ and simplify (a lot!).

   answer: $U_x^2 + U_y^2 = U_r^2 + U_\theta^2/r^2$