Math 2326, Test I

Name ______________________________

For problems 1-4, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dx} - 4y = 4x$

   answer: $y = Ce^{4x} - x - 1/4$

2. $\frac{dy}{dt} = \frac{3t^2}{4y^3+1}$, with $y(0) = 1$

   answer: $y^4 + y = t^3 + 2$

3. $\frac{dy}{dx} = \frac{3x\sqrt{1+y^2}}{y}$, with $y(0) = -1$

   answer: $y = -\sqrt{(\frac{3}{2}x^2 + \sqrt{2})^2 - 1}$

4. $\frac{dy}{dt} + 5y = 3e^{-5t}$

   answer: $y = Ce^{-5t} + 3te^{-5t}$

5. For the differential equation $y' = x + 3y, y(1) = 2$, take two steps using Euler's method with $h = 0.1$, to approximate $y(1.2)$. You may use the following table, if you want:

x	y	f(x,y) = x+3y	y + h*f(x,y)
1.0	2.00	7.00	2.70
1.1	2.70	9.20	3.62
1.2	3.62	(skip)	(skip)

6. Consider the differential equation $\frac{dP}{dt} = (\frac{P}{200} - 1)^2(100 - P^2)$

   a. Is this equation autonomous? answer: yes

   b. Is this equation separable? answer: yes

   c. Is this equation linear? answer: no

   d. Find all equilibrium points, and classify each as a source, sink or node. answer: -10 is source, 10 is sink, 200 is node

7. A 200 gallon tank is full to the brim with pure water, and 5 gallons/minute of a brine solution with 0.1 kg/gallon salt flows into it. Since the tank is full, 5 gallons/minute of well-mixed solution overflows onto the ground. If $S(t)$ is the number of kg of salt in the tank, as a function of time, find a differential equation with boundary conditions for $S(t)$ (do not solve it).

   answer: $S' = 0.5 - S/40, S(0) = 0$