Math 2326, Test I

Name ______________________________

For problems 1-4, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dx} - 4y = -12x - 5.$

   answer: $y = Ce^{4x} + 3x + 2$

2. $\frac{dy}{dt} = \frac{t}{3y^2-1}$, with $y(0) = 4$

   answer: $y^3 - y = \frac{1}{2}t^2 + 60$

3. $\frac{dy}{dx} = \frac{3x\sqrt{1+y^2}}{y}$, with $y(0) = -1$

   answer: $y = -\sqrt{(\frac{3}{2}x^2 + \sqrt{2})^2 - 1}$

4. $\frac{dy}{dt} + 6y = 5e^{-6t}.$

   answer: $y = Ce^{-6t} + 5te^{-6t}$

5. For the differential equation $y' = 3x + 2y, y(0) = 2$, take three steps using Euler's method with $h = 0.1$, to approximate $y(0.3)$. You may use the following table, if you want:

x	y	f(x,y) = 3x+2y	y + h*f(x,y)
0.0	2.000	4.000	2.400
0.1	2.400	5.100	2.910
0.2	2.910	6.420	3.552
0.3	3.552	(skip)	(skip)

6. Consider the differential equation $\frac{dP}{dt} = (\frac{P}{10} - 1)(1 - \frac{P}{5})P^2$

   a. Is this equation autonomous? (answer: yes)

   b. Is this equation linear? (answer: no)

   c. Find all equilibrium points, and classify each as a source, sink or node.
      answer: $P = 10$ is sink, $P = 5$ is source, $P = 0$ is node

7. A cup of coffee is initially at 100^o F and is left in a room with a temperature of 70^o F. Suppose that at time $t = 0$ it is cooling at a rate of 3^o F per minute. Assume that Newton's law of cooling applies, that is, the rate of cooling is proportional to the difference between the current temperature $T(t)$ and the room temperature, 70^o F. Find a formula for the temperature $T(t)$ as a function of time.

   answer: $T(t) = 70 + 30e^{-0.1t}$