Math 2326, Test I

Name ______________________________

For problems 1-4, solve the differential equations. If you cannot solve for the dependent variable, leave the solution defined implicitly. If no initial condition is given, write the general solution.

1. $\frac{dy}{dt} = 3\ sin(3t)(1+y)$.

   answer: $y(t) = De^{-cos(3t)} - 1$

2. $\frac{dy}{dt} = \frac{t}{5y^4+6y^2-4y}$, with $y(1) = 1$

   answer: $y^5 + 2y^3 - 2y^2 = \frac{1}{2}t^2 + \frac{1}{2}$

3. $\frac{dy}{dt} + 2y = 6cos(2t)$

   answer: $y(t) = Ce^{-2t} + \frac{3}{2}sin(2t) + \frac{3}{2}cos(2t)$

4. $\frac{dy}{dt} + 5y = 3e^{-5t}$, with $y(0) = 2$.

   answer: $y = 2e^{-5t} + 3te^{-5t}$

5. Find all equilibrium points of $\frac{dy}{dt} = 0.5 - cos(\pi y)$ between -1 and 1 and classify each and classify each as a source, sink or node. Then tell what happens to $y(t)$ as $t \to \infty$, if $y(0) = 0$.

answer: $y = -1/3$ is sink, $y = 1/3$ is source. $y(\infty) = -1/3$.

6. a. A cup of coffee is initially at 130^o F and is left in a room with a temperature of 80^o F. Suppose that at time $t = 0$ it is cooling at a rate of 2^o F per minute. Assume that Newton's law of cooling applies, that is, the rate of cooling is proportional to the difference between the current temperature $T(t)$ and the room temperature, 80^o F. Write a differential equation with two initial conditions for the temperature $T(t)$. (A first order differential equation usually has only one initial condition, but the second condition is to find the unknown proportionality constant.)

   answer: $T'(t) = -k(T - 80), T(0) = 130, T'(0) = -2$.

   b. Solve the differential equation you wrote in part (a), with initial conditions.

   answer: $k = 0.04, T(t) = 80 + 50e^{-0.04\ t}$.

   c. The differential equation of part (a) is autonomous and has one equilibrium point. Find the point and tell if it is a source, sink or node.

   answer $T = 80$ is a sink