Math 3323, Test I

Name ____________________________

1. If

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 1 & 6 \\ 3 & 6 & 0 & 9 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: (1,0,0,0) + x2(-2,1,0,0) + x4(-3,0,0,1)

2. Show that if Q and R are orthogonal matrices (ie, $Q^{-1} = Q^T, R^{-1} = R^T$), then QR is orthogonal also.

answer: $(QR)^{-1} = R^{-1}Q^{-1} = R^TQ^T = (QR)^T$

3. Find u^Tw and uw^T if:

$$u = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, w = \begin{bmatrix} 5 \\ 2 \\ 9 \end{bmatrix}$$

answer:

$$u^Tw = 34$$

$$uw^T = \begin{bmatrix} 5 & 2 & 9 \\ 5 & 2 & 9 \\ 15 & 6 & 27 \end{bmatrix}$$

4. Given that $1^2 + 2^2 + 3^2 + ... + n^2 = an + bn^2 + cn^3$, for all integer n, write a system of 3 equations in 3 unknowns to find a, b, c. (Don't solve the system!!)

   answer: Ax=d, where:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{bmatrix}, d = \begin{bmatrix} 1 \\ 5 \\ 14 \end{bmatrix}$$

5. Show that the vectors below are linearly dependent, by finding a, b, c not all zero, such that $au + bv + cw = 0$:

$$u = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, v = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}, w = \begin{bmatrix} 5 \\ 2 \\ 9 \end{bmatrix}$$

   answer: $u + 3v + w = 0$

6. If A is a singular square matrix, what can you say about the solution to $Ax = b$ (ie, unique solution, many solutions, no solution) if:

   a. b is the zero vector.
      answer: many solutions

   b. b is not the zero vector.
      answer: many solutions or no solution