Math 3323, Test I

Name ______________________________

1. If

$$A = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 1 & 6 \\ 3 & 6 & 0 & 9 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

find the general solution of $Ax = b$.

answer: no solution

2. Suppose v_1, v_2, v_3 are mutually orthogonal nonzero vectors, that is, $v_1^T v_2 = v_1^T v_3 = v_2^T v_3 = 0$. Show that the set of vectors v_1, v_2, v_3 is linearly independent.

answer: if $av_1 + bv_2 + cv_3 = 0, 0 = v_1^T(av_1 + bv_2 + cv_3) = a\|v_1\|^2$ and since $\|v_1\| \neq 0, a = 0$. Similarly, b=c=0.

3. Find AB and BA if:

$$A = \begin{bmatrix} 1 & 5 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}$$

answer:

$$AB = \begin{bmatrix} 15 & 17 \\ 9 & 8 \end{bmatrix}, BA = \begin{bmatrix} 7 & 29 \\ 5 & 16 \end{bmatrix}$$

4. Find the polynomial of degree 2 or less, $y = a_0 + a_1 x + a_2 x^2$ which passes through the points $(0, 3), (1, 5), (2, 9)$. Solve the linear system by hand, and show your work.

   answer: $y = 3 + x + x^2$

5. Determine if the vectors below are linearly dependent or not by trying to find a, b, c such that $au + bv + cw = 0$:

$$u = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, v = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

   answer: $2u + v - w = 0$ so dependent

6. If A is a singular square matrix, what can you say about the solution to $Ax = b$ (ie, unique solution, many solutions, no solution) if:

   a. b is the zero vector.
      answer: many solutions

   b. b is not the zero vector.
      answer: many solutions or no solution