Math 3323, Final

Name _____

1. If A =
$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

 a. Find all eigenvalues of A.

 answer: $\lambda = 2, 3$

 b. For each eigenvalue, find a basis for the subspace of eigenvectors (the eigenspace).

 answer: for $\lambda = 2$, basis is $[(1,0,0), (0,-2,1)]$, for $\lambda = 3$, basis is $[(1,1,0)]$.

2. Find the eigenvalues of A =

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

 answer: $5, 3, 3 + \sqrt{10}i, 3 - \sqrt{10}i$.

3. Find the determinant of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 5 & 2 & 3 \\ 0 & 3 & 1 \end{bmatrix}$$

 answer: $\det(A) = 18$

4. Find the general solution of (Hint: see problem 1):

$$\frac{dx}{dt} = 2x + y + 2z$$

$$\frac{dy}{dt} = 3y + 2z$$

1

$$\frac{dz}{dt} = 2z$$

answer:
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C_1 e^{3t} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_3 e^{2t} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

5. Find the general solutions of the following linear systems:

a.
$$\begin{array}{rcl} x \quad -4y \quad +2z &=& 0 \\ 3x \quad -12y \quad +4z &=& 7 \\ -2x \quad +8y \quad +2z &=& -21 \end{array}$$

   answer: $(7,0,-3.5) + y(4,1,0)$

b.
$$\begin{array}{rcl} x \quad -4y \quad +2z &=& 0 \\ 3x \quad -12y \quad +4z &=& 7 \\ -2x \quad +8y \quad +2z &=& 7 \end{array}$$
   answer: no solution

6. If $A =$

$$\begin{bmatrix} 1 & 4 & 2 & -3 \\ 3 & 10 & 0 & -2 \\ 2 & 6 & -2 & 1 \end{bmatrix}$$

What is the rank of A? The dimension of the range of A? What is the dimension of the null space of A?

answer: reduced echelon form is:

$$\begin{bmatrix} 1 & 0 & -10 & 11 \\ 0 & 1 & 3 & -3.5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so rank = 2, dim(range) = rank = 2, dim(null space) = 4 - rank = 2

7. Find a basis for the null space of the matrix A of problem 6.

answer: [(10,-3,1,0),(-11,3.5,0,1)]

8. Write the equations for:

    a. The line through the points (1,-1,2) and (3,4,3).
       answer: $x = 1 + 2t, y = -1 + 5t, z = 2 + t$

    b. The plane through (1,-1,2), perpendicular to this line.
       answer: $2(x - 1) + 5(y + 1) + (z - 2) = 0$

9. Which of the following subsets are subspaces? For those that are subspaces, give the dimension.

    a. The null space of the matrix

    $$A = \begin{bmatrix} 1 & 1 & 3 & 0 & -1 \\ 2 & 2 & 6 & 0 & -2 \\ 3 & 3 & 9 & 0 & -3 \end{bmatrix}$$

    answer: yes, dimension=4

    b. The row space of A.
       answer: yes, dimension=1

    c. The range of A.
       answer: yes, dimension=1

    d. The line in R^3: $x = 1 + t, y = 2t, z = -t$.
       answer: no

10. Find the least-squares linear fit $y = b + mx$ to the points $(-1, -1), (0, 1), (1, 2), (2, 4)$.
    answer: $y = 0.7 + 1.6x$