

Math 3323, Final

Name _____

1. Find the eigenvalues and corresponding eigenvectors for $A =$

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

answer: $\lambda_1 = 1 + 2i$, $x_1 = (i, 1)$ or $(1, -i)$; $\lambda_2 = 1 - 2i$, $x_2 = (-i, 1)$ or $(1, i)$.

2. Find the general solution of:

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = 6x - y$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-3t} \begin{bmatrix} 1 \\ -3 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

3. Find a basis for the subspace spanned by the four vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

answer: $[(1, 2, 1), (0, 1, -2)]$ (other answers possible)

4. Find the determinant of $A =$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

answer: 80

5. Which of the following subsets are subspaces? For those that are subspaces, give the dimension.

a. The null space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 0 \\ 2 & 1 & 5 & 4 & 0 \\ 1 & 2 & 4 & -1 & 0 \end{bmatrix}$$

answer: yes, dimension=3

b. The row space of A.

answer: yes, dimension=2

c. The range of A.

answer: yes, dimension=2

d. The plane $x+y+z=2$.

answer: no

6. If $A =$

$$\begin{bmatrix} 3 & -1 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

a. Find all eigenvalues of A.

answer: $\lambda = 1, 5$

b. For each eigenvalue, find a basis for the subspace of eigenvectors (the eigenspace).

answer: for $\lambda = 1$, basis is $[(1, 2, 0)]$, for $\lambda = 5$, basis is $[(1, -2, 0), (0, 0, 1)]$.

7. If

$$u = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix},$$

a. Find $u \times v$.

answer: $\langle 8, 4, 2 \rangle$

b. Find the equation of the plane through $(2, 1, 2)$ parallel to the vectors u and v .

answer: $8x + 4y + 2z = 24$

- c. Find the equations of a line through $(2, 1, 2)$ perpendicular to the plane in part b.
answer: $x = 2 + 8t, y = 1 + 4t, z = 2 + 2t$
- d. Find the area of a parallelogram with edges u and v .
answer: $\sqrt{84}$
8. Find the least-squares linear fit $y = b + mx$ to the points $(0, 1), (1, 2), (2, 4)$.
answer: $y = (5 + 9x)/6$