Taylor series for $f(x)$ expanded around the point a:

$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + ... + \frac{1}{n!}f^{(n)}(a)(x-a)^n$

$+\frac{1}{(n+1)!}f^{(n+1)}(a)(x-a)^{n+1} + ...$

or

$f(x) = T_n(x) + E_n(x)$

where

$E_n(x) = \frac{1}{(n+1)!}f^{(n+1)}(a)(x-a)^{n+1} + ...$

or

$E_n(x) = \frac{1}{(n+1)!}f^{(n+1)}(\psi)(x-a)^{n+1}$, with ψ somewhere between x and a

Often we will write the Taylor series as:

$f(a+h) = f(a) + f'(a)h + \frac{1}{2!}f''(a)h^2 + \frac{1}{3!}f'''(a)h^3 + ... + \frac{1}{n!}f^{(n)}(a)h^n + ...$

where $h \equiv x - a$. Or, more often:

$f(x+h) = f(x) + f'(x)h + \frac{1}{2!}f''(x)h^2 + \frac{1}{3!}f'''(x)h^3 + ... + \frac{1}{n!}f^{(n)}(x)h^n + ...$

Example: If $f(x) = e^x$ and $a = 0$:

$T_n(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + ... + \frac{1}{n!}x^n$

and

$E_n(x) = \frac{1}{(n+1)!}f^{(n+1)}(\psi)x^{n+1} = \frac{1}{(n+1)!}e^{\psi}x^{n+1}$

If $-100 < x < 100$, the error in T_n is bounded by:

$|E_n(x)| < \frac{1}{(n+1)!}e^{100}100^{n+1}$

Thus $n = 700$ is sufficient to ensure an extremely small error for any x in this range, because

$|E_{700}(x)| < \frac{1}{701!}e^{100}100^{701} < 10^{-246}$

but, when x is negative, severe roundoff error can occur:

$T_n(-x) = 1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5 + ...$
$= [1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + ...] - [x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + ...]$
$= cosh(x) - sinh(x)$

for example, when $-x = -50$:

$T_n(-50) = cosh(50) - sinh(50) =$

$0.25923527642 * 10^{22} - 0.25923527642 * 10^{22} = 0.19287498479 * 10^{-21}$