Math 4329, Test I

Name _____

1. a. If $f(x) = x^4 - 10x^3 + x + 3$, find the Taylor polynomial $T_2(x)$ of degree 2 which matches f, f' and f'' at $a = 1$.

   b. Use the Taylor remainder formula to get a reasonable bound on the maximum of the error $|f(x) - T_2(x)|$ for $0.5 < x < 1.5$.

2. IEEE double precision floating point numbers are stored with one sign bit (0 for positive numbers), 11 exponent bits ($1023 + e$ is stored in binary form here, where e is the exponent of 2), and then 52 bits for the mantissa. Show exactly how 131.125_{10} would be stored (in either binary or hexadecimal form).

3. If $f(x) = (x-5)^m$ (m is a positive integer) and Newton's method is used to find the root r=5, this can be thought of as a fixed point iteration $x_{n+1} = g(x_n)$. Calculate $g'(5)$ and tell what values of m will cause Newton's method to converge linearly, and what values will cause it to converge even faster. For what range of starting values x_0 will Newton's method converge, if m=3?

4. Estimate the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $10^{-2}, 10^{-5}$ and 10^{-10}.

5. Show how Newton's method could be used to find $b^{\frac{1}{5}}$ without doing anything other than add, subtract, multiply and divide.

6. Write out the secant iteration for finding a root of $f(x) = \frac{1}{x} - b$, where no divisions are done, thus it could be used to find the root $\frac{1}{b}$ on a computer that can't divide.