Math 4329, Test II

Name _____

1. a. A table of values for $f(x)$ is:

x	$f(x)$
1.0	0.1
1.1	0.0
1.2	0.0
1.3	0.0

Use cubic interpolation to estimate $f(1.05)$.

b. If it is known that $|f^{iv}(x)| < 0.001$ for all $1 < x < 1.3$, obtain as small a bound as possible on the error in your estimate of $f(1.05)$.

2. Use Taylor series expansions to determine the error (not just the order of the error) in the approximation $u''(t) \approx \frac{u(t+h)-2u(t)+u(t-h)}{h^2}$

3. Determine the value for r which makes

   $\int_0^h f(x)dx \approx \frac{1}{3}hf(rh) + \frac{1}{3}hf(\frac{h}{2}) + \frac{1}{3}hf((1-r)h)$

   as high order as possible. Is this the Gauss 3 point formula?

4. (a) If we fit a polynomial of degree N through N+1 points, then change the y value at one point, where does the polynomial change, in general? (b) What about a natural cubic spline interpolant, same question?

5. (a) What boundary conditions does a natural cubic spline satisfy? (b) Give a scenario where this spline is a better idea than a "not-a-knot" cubic spline. (c) Give a scenario where it is not as good.

6. Below is a MATLAB program to solve Ax=b with no pivoting

```
function x = geln(A,b,n)
% forward elimination
for k=1:n-1
   if abs(A(k,k)) == 0
      error('Zero pivot encountered')
   end
   for i=k+1:n
      amul = -A(i,k)/A(k,k);
% add amul times row k to row i
      for j=k:n
         A(i,j) = A(i,j) + amul*A(k,j);
      end
      b(i) = b(i) + amul*b(k);
   end
end
% back substitution
if A(n,n) == 0
   error('Zero pivot encountered')
end
x(n) = b(n)/A(n,n);
for k=n-1:-1:1
   sum = 0;
   for j=k+1:n
      sum = sum + A(k,j)*x(j);
   end
   x(k) = (b(k)-sum)/A(k,k);
end
```

(a) Indicate which line in this program will account for nearly 100% of the computer time, when n is large. (b) Approximately how many multiplications are done during the forward elimination? (c) What about the back substitution, same question? (d) Approximately how much would the computer time increase if pivoting were done? (Assume n is large still.)