Math 5330 Final Exam

Name ______________________________

1. a. If the primal LP problem is:
maximize $P = c^T x$, with constraints $Ax = b$ and bounds $x \geq 0$
And the dual is:
minimize $D = b^T y$, with constraints $A^T y \geq c$
Show that, for any primal-feasible x and any dual-feasible $y, b^T y \geq c^T x$.

   b. If the primal problem is unbounded (maximum is infinite) what can we say about the dual problem? What is known about the maximum P of the primal problem and the minimum D of the dual?

2. a. Find the straight line $y = mx + b$ which most closely fits the data points $(0, 1), (1, 4), (2, 5)$ in the L_2 norm.

b. Write a linear programming problem which, if solved (but don't solve it), would produce the straight line $y = mx + b$ which most closely fits these same data points in the L_∞ norm. Write the constraints in the form $Ax \geq b$. (Hint: you will have 3 unknowns, m, b and ϵ, and 3 constraints involving absolute values, which translate into 6 linear constraints.)

3. Two factories have 180 and 150 cars, two dealers need 100 and 60 delivered to them. The cost C_{ij} to transport each car from factory i to dealer j is: $C_{11} = 100, C_{12} = 150, C_{21} = 250, C_{22} = 240$. Set up the initial simplex tableaux for this problem (but don't solve!), using slack and artificial variables as needed.

4. Use the simplex method to solve

a. max $P = 2x + 3y$
with

$$\begin{aligned} x + y &\leq 4 \\ x + 2y &\leq 6 \end{aligned}$$

and $x, y \geq 0$

b. max $P = 3x_1 + x_2 + 2x_3$
with

$$\begin{aligned} -2x_1 + x_2 + x_3 &\leq 2 \\ 2x_2 + x_3 &\leq 6 \end{aligned}$$

and $x_1, x_2, x_3 \geq 0$

5. What is the order of work for each of the following? Assume all matrices are N by N and full unless otherwise stated, and assume advantage is taken of any special structure mentioned.

   a. One iteration of the Jacobi method (to zero one off-diagonal element) to find the eigenvalues of a symmetric matrix A.

   b. Solution of $Ax = b$ using Gaussian elimination, if A is upper Hessenberg.

   c. One QR iteration, if A is symmetric and tridiagonal.

   d. Solution of min $\|Ax - b\|_2$ using orthogonal reduction, where A is M by N, and $M >> N$.

   e. One simplex step, for solving max $c^T x$ with $Ax \leq b, x \geq 0$, where A is M by N, and $N >> M$.

   f. Solution of $Ax = b$ if an LU decomposition is known.

   g. One iteration of the inverse power method, for finding the smallest eigenvalue of tridiagonal matrix A.

   h. Solution of $Ax = b$ using Gaussian elimination, if A is banded, with bandwidth $N^{\frac{1}{3}}$.

   i. The solution of $Ax = b$ using Gaussian elimination, if A is tridiagonal except A_{1N} and A_{N1} are also nonzero.

   j. One Gauss-Seidel iteration, for solving $Ax = b$.